INTRODUCTION TO STELLAR STATISTICS

BY

C. V. L. CHARLIER

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CHAPTER I.

APPARENT ATTRIBUTES OF THE STARS.

1. Our knowledge of the stars is based on their *apparent* attributes, obtained from the astronomical observations. The object of astronomy is to deduce herefrom the real or *absolute* attributes of the stars, which are their position in space, their movement, and their physical nature.

The apparent attributes of the stars are studied by the aid of their *radiation*. The characteristics of this radiation may be described in different ways, according as the nature of the light is defined. (Undulatory theory, Emission theory.)

From the statistical point of view it will be convenient to consider the radiation as consisting of an emanation of small particles from the radiating body (the star). These particles are characterized by certain attributes, which may differ in degree from one particle to another. These attributes may be, for instance, the diameter and form of the particles, their mode of rotation, &c. By these attributes the optical and electrical properties of the radiation are to be explained. I shall not here attempt any such explanation, but shall confine myself to the property which the particles have of possessing a different mode of deviating from the rectilinear path as they pass from one medium to another. This deviation depends in some way on one or more attributes of the particles. Let us suppose that it depends on a single attribute, which, with a terminology derived from the undulatory theory of HUYGHENS, may be called the *wave-length* (λ) of the particle.

The statistical characteristics of the radiation are then in the first place:—

(1) the total number of particles or the *intensity* of the radiation;

(2) the *mean wave-length* (λ_0) of the radiation, also called (or nearly identical with) the *effective* wave-length or the colour;

(3) *the dispersion of the wave-length.* This characteristic of the radiation may be determined from the *spectrum,* which also gives the variation of the radiation with λ, and hence may also determine the mean wave-length of the radiation.

Moreover we may find from the radiation of a star its apparent place on the sky.

The intensity, the mean wave-length, and the dispersion of the wave-length are in a simple manner connected with the *temperature* (T) of the star. According to the radiation laws of STEPHAN and WIEN we find, indeed (compare L. M. 41[1]) that the intensity is proportional to the fourth power of T, whereas the mean wave-length and the dispersion of the wave-length are both inversely proportional to T. It follows that with increasing temperature the mean wave-length diminishes—the colour changing into violet—and simultaneously the dispersion of the wave-length and also even the total length of the spectrum are reduced (decrease).

2. *The apparent position of a star* is generally denoted by its right ascension (α) and its declination (δ). Taking into account the apparent distribution of the stars in space, it is, however, more practical to characterize the position of a star by its galactic longitude (l) and its galactic latitude (b). Before defining these coordinates, which will be generally used in the following pages, it should be pointed out that we shall also generally give the coordinates α and δ of the stars in a particular manner. We shall therefore use an abridged notation, so that if for instance $\alpha = 17^h\, 44^m\!.7$ and $\delta = +35^\circ\!.84$, we shall write

$$(\alpha\, \delta) = (174435).$$

If δ is negative, for instance $\delta = -35^\circ\!.84$, we write

$$(\alpha\, \delta) = (1744\mathit{35}),$$

so that the last two figures are in italics.

This notation has been introduced by PICKERING for variable stars and is used by him everywhere in the Annals of the Harvard

[1]) Meddelanden från Lunds observatorium, No. 41.

Observatory, but it is also well suited to all stars. This notation gives, simultaneously, the characteristic *numero* of the stars. It is true that two or more stars may in this manner obtain the same characteristic *numero*. They are, however, easily distinguishable from each other through other attributes.

The *galactic* coordinates l and b are referred to the Milky Way (the Galaxy) as plane of reference. The pole of the Milky Way has according to HOUZEAU and GOULD the position $(\alpha\delta) = (124527)$. From the distribution of the stars of the spectral type B I have in L. M. II, 14[1]) found a somewhat different position. But having ascertained later that the real position of the galactic plane requires a greater number of stars for an accurate determination of its value, I have preferred to employ the position used by PICKERING in the Harvard catalogues, namely $(\alpha\delta) = (124028)$, or

$$\alpha = 12^h\,40^m = 190°, \quad \delta = +28°,$$

which position is now exclusively used in the stellar statistical investigations at the Observatory of Lund and is also used in these lectures.

The galactic longitude (l) is reckoned from the ascending node of the Milky Way on the equator, which is situated in the constellation *Aquila*. The galactic latitude (b) gives the angular distance of the star from the Galaxy. On plate I, at the end of these lectures, will be found a fairly detailed diagram from which the conversion of α and δ of a star into l and b may be easily performed. All stars having an apparent magnitude brighter than 4^m are directly drawn.

Instead of giving the galactic longitude and latitude of a star we may content ourselves with giving the galactic *square* in which the star is situated. For this purpose we assume the sky to be divided into 48 squares, all having the same surface. Two of these squares lie at the northern pole of the Galaxy and are designated GA_1 and GA_2. Twelve lie north of the galactic plane, between 0° and 30° galactic latitude, and are designated $GC_1, GC_2, \ldots, GC_{12}$. The corresponding squares south of the galactic equator (the plane of the Galaxy) are called $GD_1, GD_2, \ldots, GD_{12}$. The two polar squares at the south pole are called GF_1 and GF_2. Finally we have 10 B-squares, between the A- and C-squares and 10 corresponding E-squares in the southern hemisphere.

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1) Meddelanden från Lunds Astronomiska Observatorium, Serie II, No. 14.

The distribution of the squares in the heavens is here graphically represented in the projection of FLAMSTEED, which has the advantage of giving areas proportional to the corresponding spherical areas, an arrangement necessary, or at least highly desirable, for all stellar statistical researches. It has also the advantage of affording a continuous representation of the whole sky.

The correspondence between squares and stellar constellations is seen from plate II. Arranging the constellations according to their galactic longitude we find north of the galactic equator (in the C-squares) the constellations:—

Hercules, Cygnus, Cepheus, Cassiopæa, Auriga, Gemini, Canis Minor, Pyxis, Vela, Centaurus, Scorpius, Ophiochus,

and south of this equator (in the D-squares):—

Aquila, Cygnus, Lacerta, Andromeda, Perseus, Orion, Canis Major, Puppis, Carina, Circinus, Corona australis, Sagittarius,

mentioning only one constellation for each square.

At the north galactic pole (in the two A-squares) we have:—

Canes Venatici and Coma Berenicis,

and at the south galactic pole (in the two F-squares):—

Cetus and Sculptor.

3. *Changes in the position of a star.* From the positions of a star on two or more occasions we obtain its apparent motion, also called the *proper motion* of the star. We may distinguish between a *secular* part of this motion and a *periodical* part. In both cases the motion may be either a reflex of the motion of the observer, and is then called *parallactic* motion, or it may be caused by a real motion of the star. From the parallactic motion of the star it is possible to deduce its distance from the sun, or its parallax. The periodic parallactic proper motion is caused by the motion of the earth around the sun, and gives the *annual parallax* (π). In order to obtain available annual parallaxes of a star it is usually necessary for the star to be nearer to us than 5 siriometers, corresponding to a parallax greater than $0''.04$. More seldom we may in this manner obtain trustworthy values for a distance amounting

to 10 siriometers ($\pi = 0''.02$), or even still greater values. For such large distances the *secular* parallax, which is caused by the progressive motion of the sun in space, may give better results, especially if the mean distance of a group of stars is simultaneously determined. Such a value of the secular parallax is also called, by KAPTEYN, the *systematic* parallax of the stars.

When we speak of the proper motion of a star, without further specification, we mean always the secular proper motion.

4. Terrestial distances are now, at least in scientific researches, universally expressed in kilometres. A kilometre is, however, an inappropriate unit for celestial distances. When dealing with distances in our planetary system, the astronomers, since the time of NEWTON, have always used the mean distance of the earth from the sun as universal unit of distance. Regarding the distances in the stellar system the astronomers have had a varying practice. German astronomers, SEELIGER and others, have long used a stellar unit of distance corresponding to an annual parallax of $0''.2$, which has been called a "Siriusweite". To this name it may be justly objected that it has no international use, a great desideratum in science. Against the theoretical definition of this unit it may also be said that a distance is suitably to be defined through another distance and not through an angle—an angle which corresponds moreover, in this case, to the *harmonic* mean distance of the star and not to its arithmetic mean distance. The same objection may be made to the unit "parsec." proposed in 1912 by TURNER.

For my part I have, since 1911, proposed a stellar unit which, both in name and definition, nearly coincides with the proposition of SEELIGER, and which will be exclusively used in these lectures. A *siriometer* is put equal to 10^6 times the planetary unit of distance, corresponding to a parallax of $0''.206265$ (in practice sufficiently exactly $0''.2$).

In popular writings, another unit: a *light-year*, has for a very long time been employed. The relation between these units is

$$1 \text{ siriometer} = 15.79 \text{ light-years},$$
$$1 \text{ light-year} = 0.0633 \text{ siriometers}.$$

5. In regard to *time* also, the terrestrial units (second, day, year) are too small for stellar wants. As being consistent with the unit of

distance, I have proposed for the stellar unit of time a *stellar year* (st.), corresponding to 10^6 years. We thus obtain the same relation between the stellar and the planetary units of length and time, which has the advantage that a *velocity* of a star expressed in siriometers per stellar year is expressed with the same numerals in planetary units of length per year.

Spectroscopic determinations of the velocities, through the DOPPLER-principle, are generally expressed in km. per second. The relation with the stellar unit is the following:

$$1 \text{ km./sec.} = 0.2111 \text{ sir./st.,}$$
$$= 0.2111 \text{ planetary units per year,}$$
$$1 \text{ sir./st.} = 4.7375 \text{ km./sec.}$$

Thus the velocity of the sun is 20 km./sec. or 4.22 sir./st. (= 4.22 earth distances from the sun per year).

Of the numerical value of the stellar velocity we shall have opportunity to speak in the following. For the present it may suffice to mention that most stars have a velocity of the same degree as that of the sun (in the mean somewhat greater), and that the highest observed velocity of a star amounts to 72 sir./st. (= 340 km./sec.). In the next chapter I give a table containing the most speedy stars. The least value of the stellar velocity is evidently equal to zero.

6. *Intensity of the radiation.* This varies within wide limits. The faintest star which can give an impression on the photographic plates of the greatest instrument of the Mount Wilson observatory (100 inch reflector) is nearly 100 million times fainter than Sirius, a star which is itself more than 10000 million times fainter than the sun—speaking of apparent radiation.

The intensity is expressed in *magnitudes* (m). The reason is partly that we should otherwise necessarily have to deal with very large numbers, if they were to be proportional to the intensity, and partly that it is proved that the human eye apprehends quantities of light as proportional to m.

This depends upon a general law in psycho-physics, known as FECHNER's *law*, which says that changes of the apparent impression of light are proportional not to the changes of the intensity but to these changes divided by the primitive intensity. A similar law is valid for all sensations. A conversation is inaudible in the vicinity of a waterfall.

An increase of a load in the hand from nine to ten hectograms makes no great difference in the feeling, whereas an increase from one to two hectograms is easily appreciable. A match lighted in the day-time makes no increase in the illumination, and so on.

A mathematical analysis shows that from the law of FECHNER it follows that the impression increases in *arithmetical* progression (1, 2, 3, 4, . . .) simultaneously with an increase of the intensity in *geometrical* progression (I, I^2, I^3, I^4, . . .). It is with the sight the same as with the hearing. It is well known that the numbers of vibrations of the notes of a harmonie scale follow each other in a geometrical progression though, for the ear, the intervals between the notes are apprehended as equal. The magnitudes play the same rôle in relation to the quantities of light as do the logarithms to the corresponding numbers. If a star is considered to have a brightness intermediate between two other stars it is not the *difference* but the *ratio* of the quantities of light that is equal in each case.

The branch of astronomy (or physics) which deals with intensities of radiation is called *photometry*. In order to determine a certain scale for the magnitudes we must chose, in a certain manner, the *zero-point* of the scale and the *scale-ratio*.

Both may be chosen arbitrarily. The *zero-point* is now almost unanimously chosen by astronomers in accordance with that used by the Harvard Observatory. No rigorous definition of the Harvard zero-point, as far as I can see, has yet been given (compare however H. A. 50[1]), but considering that the Pole-star (α Ursæ Minoris) is used at Harvard as a fundamental star of comparison for the brighter stars, and that, according to the observations at Harvard and those of HERZSPRUNG (A. N. 4518 [1911]), the light of the Pole-star is very nearly invariable, we may say that *the zero-point of the photometric scale is chosen in such a manner that for the Pole-star $m = 2.12$.* If the magnitudes are given in another scale than the Harvard-scale (H. S.), it is necessary to apply the zero-point correction. This amounts, for the Potsdam catalogue, to $-0^m.16$.

It is further necessary to determine the *scale-ratio*. Our magnitudes for the stars emanate from PTOLEMY. It was found that the scale-ratio—

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[1]) Annals of the Harvard Observatory, vol. 50.

giving the ratio of the light-intensities of two consecutive classes of magnitudes—according to the older values of the magnitudes, was approximately equal to $2^{1}/_{2}$. When exact photometry began (with instruments for measuring the magnitudes) in the middle of last century, the scale-ratio was therefore put equal to 2.5. Later it was found more convenient to choose it equal to 2.512, the logarithm of which number has the value 0.4. The magnitudes being themselves logarithms of a kind, it is evidently more convenient to use a simple value of the logarithm of the ratio of intensity than to use this ratio itself. This scale-ratio is often called the POGSON-scale (used by POGSON in his "Catalogue of 53 known variable stars", Astr. Obs. of the Radcliffe Observatory, 1856), and is now exclusively used.

It follows from the definition of the scale-ratio that two stars for which the light intensities are in the ratio 100 : 1 differ by exactly 5 magnitudes. A star of the 6^{th} magnitude is 100 times fainter than a star of the first magnitude, a star of the 11^{th} magnitude 10000 times, of the 16^{th} magnitude a million times, and a star of the 21^{st} magnitude 100 million times fainter than a star of the first magnitude. The star magnitudes are now, with a certain reservation for systematic errors, determined with an accuracy of $0^{\mathrm{m}}.1$, and closer. Evidently, however, there will correspond to an error of 0.1 in the magnitude a considerable uncertainty in the light ratios, when these differ considerably from each other.

Sun	$-26^{\mathrm{m}}.60$
Full moon ...	$-11^{\mathrm{m}}.77$
Venus	$-4^{\mathrm{m}}.28$
Jupiter	$-2^{\mathrm{m}}.35$
Mars	$-1^{\mathrm{m}}.79$
Mercury	$-0^{\mathrm{m}}.90$
Saturn	$+0^{\mathrm{m}}.88$
Uranus	$+5^{\mathrm{m}}.86$
Neptune	$+7^{\mathrm{m}}.66$

A consequence of the definition of m is that we also have to do with *negative* magnitudes (as well as with negative logarithms). Thus, for example, for *Sirius* $m = -1.58$. The magnitudes of the greater planets, as well as those of the moon and the sun, are also negative, as will be seen from the adjoining table, where the values are taken from "Die Photometrie der Gestirne" by G. MÜLLER.

The apparent magnitude of the sun is given by ZÖLLNER (1864). The other values are all found in Potsdam, and allude generally to the maximum value of the apparent magnitude of the moon and the planets.

The brightest star is *Sirius,* which has the magnitude $m = -1.58$. The magnitude of the faintest visible star evidently depends on the penetrating power of the instrument used. The telescope of WILLIAM

HERSCHEL, used by him and his son in their star-gauges and other stellar researches, allowed of the discerning of stars down to the 14th magnitude. The large instruments of our time hardly reach much farther, for visual observations. When, however, photographic plates are used, it is easily possible to get impressions of fainter stars, even with rather modest instruments. The large 100-inch mirror of the Wilson Observatory renders possible the photographic observations of stars of the 20th apparent magnitude, and even fainter.

The observations of visual magnitudes are performed almost exclusively with the photometer of ZÖLLNER in a more or less improved form.

7. *Absolute magnitude.* The apparent magnitude of a star is changed as the star changes its distance from the observer, the intensity increasing indirectly as the square of the distance of the star. In order to make the magnitudes of the stars comparable with each other it is convenient to reduce them to their value at a certain unit of distance. As such we choose one siriometer. The corresponding magnitude will be called the *absolute* magnitude and is denoted by M.[1]) We easily find from the table given in the preceding paragraph that the absolute magnitude of the sun, according to ZÖLLNER's value of m, amounts to $+3.4$, of the moon to $+31.2$. For Jupiter we find $M = +24.6$, for Venus $M = +25.3$. The other planets have approximately $M = +30$.

For the absolute magnitudes of those stars for which it has hitherto been possible to carry out a determination, we find a value of M between -8 and $+13$. We shall give in the third chapter short tables of the absolutely brightest and faintest stars now known.

8. *Photographic magnitudes.* The magnitudes which have been mentioned in the preceding paragraphs all refer to observations taken with the eye, and are called *visual* magnitudes. The total intensity of a star is, however, essentially dependent on the instrument used in measuring the intensity. Besides the eye, the astronomers use a photographic plate, a bolometer, a photo-electric cell, and other instruments. The

[1]) In order to deduce from M the apparent magnitude at a distance corresponding to a parallax of 1″ we may subtract $3^{m}.43$. To obtain the magnitude corresponding to a parallax of 0″.1 we may add 1.57. The latter distance is chosen by some writers on stellar statistics.

difference in the results obtained with these instruments is due to the circumstance that different parts of the radiation are taken into account.

The usual photographic plates, which have their principal sensibility in the violet parts of the spectrum, give us the *photographic* magnitudes of the stars. It is, however, to be remarked that these magnitudes may vary from one plate to another, according to the distributive function of the plate (compare L. M. 67). This variation, which has not yet been sufficiently studied, seems however to be rather inconsiderable, and must be neglected in the following.

The photographic magnitude of a star will in these lectures be denoted by m', corresponding to a visual magnitude m.

In practical astronomy use is also made of plates which, as the result of a certain preparation (in colour baths or in other ways), have acquired a distributive function nearly corresponding to that of the eye, and especially have a maximum point at the same wave-lengths. Such magnitudes are called *photo-visual* (compare the memoir of PARKHURST in A. J. 36 [1912]).

The photographic magnitude of a star is generally determined from measurements of the diameter of the star on the plate. A simple mathematical relation then permits us to determine m'. The diameter of a star image increases with the time of exposure. This increase is due in part to the diffraction of the telescope, to imperfect acromasie or spherical aberration of the objective, to irregular grinding of the glass, and especially to variations in the refraction of the air, which produce an oscillation of the image around a mean position.

The *zero-point* of the photographic magnitudes is so determined that this magnitude coincides with the visual magnitude for such stars as belong to the spectral type A 0 and have $m = 6.0$, according to the proposal of the international solar conference at Bonn, 1911.

Determinations of the photographic or photo-visual magnitudes may now be carried out with great accuracy. The methods for this are many and are well summarised in the Report of the Council of the R. A. S. of the year 1913. The most effective and far-reaching method seems to be that proposed by SCHWARZSCHILD, called the half-grating method, by which two exposures are taken of the same part of the sky, while at one of the exposures a certain grating is used that reduces the magnitudes by a constant degree.

9. *Colour of the stars.* The radiation of a star is different for different wave-lengths (λ). As regarding other mass phenomena we may therefore mention:—(1) the *total radiation* or intensity (I), (2) the *mean wave-length* (λ_0), (3) the *dispersion of the wave-length* (σ). In the preceding paragraphs we have treated of the total radiation of the stars as this is expressed through their magnitudes. The mean wave-length is pretty closely defined by the *colour,* whereas the dispersion of the wave-length is found from the *spectrum* of the stars.

There are blue (B), white (W), yellow (Y) and red (R) stars, and intermediate colours. The exact method is to define the colour through the mean wave-length (and not conversely) or the *effective* wave-length as it is most usually called, or from the *colour-index.* We shall revert later to this question. There are, however, a great many direct eye-estimates of the colour of the stars.

Colour corresponding to a given spectrum.

Sp.	*Colour*	*Number*
B 3	YW^-	161
A 0	YW^-	788
A 5	YW	115
F 5	YW, WY^-	295
G 5	WY	216
K 5	WY^+, Y^-	552
M	Y, Y^+	95
	Sum . . .	2222

Spectrum corresponding to a given colour.

Colour	*Sp.*	*Number*
W, W^+	A 0	281
YW^-	A 0	356
YW	A 5	482
YW^+, YW^-	F 3	211
WY	G 4	264
WY^+, Y^-	K 1	289
Y, Y^+	K 4	254
RY^-, RY	K 5	85
	Sum . . .	2222

The signs + and — indicate intermediate shades of colour.

The preceding table drawn up by Dr. MALMQUIST from the colour observations of MÜLLER and KEMPF in Potsdam, shows the connection between the colours of the stars and their spectra.

The Potsdam observations contain all stars north of the celestial equator having an apparent magnitude brighter than $7^m.5$.

We find from these tables that there is a well-pronounced *regression* in the correlation between the spectra and the colours of the stars. Taking together all white stars we find the corresponding mean spectral type to be A0, but to A0 corresponds, upon an average, the colour yellow-white. The yellow stars belong in the mean to the K-type, but the K-stars have upon an average a shade of white in the yellow colour. The coefficient of correlation (r) is not easy to compute in this case, because one of the attributes, the colour, is not strictly graduated (*i. e.* it is not expressed in numbers defining the colour).[1] Using the coefficient of contingency of PEARSON, it is, however, possible to find a fairly reliable value of the coefficient of correlation, and MALMQUIST has in this way found $r = +0.85$, a rather high value.

In order to facilitate the discussion of the relation between colour and spectrum it is convenient to deal here with the question of the spectra of the stars.

10. *Spectra of the stars.* In order to introduce the discussion I first give a list of the wave-lenghts of the FRAUENHOFER lines in the spectrum, and the corresponding chemical elements.

FRAUENHOFER line	*Element*	λ
A		759.4
B		686.8
C (α)	H (hydrogen)	656.3
D_1	Na (Sodium)	589.6
D_3	He	587.6
E	Fe (iron)	527.0
F (β)	H	486.2
(γ)	H	434.1
G	Ca (calcium)	430.8
h (δ)	H	410.2
H (ε)	Ca (H)	396.9
K	Ca	393.4

[1]) The best colour-scale of the latter sort seems to be that of OSTHOFF.

The first column gives the FRAUENHOFER denomination of each line. Moreover the hydrogen lines α, β, γ, δ, ε are denoted. The second column gives the name of the corresponding element, to which each line is to be attributed. The third column gives the wave-length expressed in millionths of a millimeter as unit ($\mu\mu$).

On plate III, where the classification of the stellar spectra according to the Harvard system is reproduced, will be found also the wave-lengths of the principal H and He lines.

By the visual spectrum is usually understood the part of the radiation between the FRAUENHOFER lines A to H ($\lambda = 760$ to $400\ \mu\mu$), whereas the photographic spectrum generally lies between F and K ($\lambda = 500$ to $400\ \mu\mu$).

In the earliest days of spectroscopy the spectra of the stars were classified according to their visual spectra. This classification was introduced by SECCHI and was later more precisely defined by VOGEL. The three classes I, II, III of VOGEL correspond approximately to the colour classification into white, yellow, and red stars. Photography has now almost entirely taken the place of visual observations of spectra, so that SECCHI's and VOGEL's definitions of the stellar spectra are no longer applicable. The terminology now used was introduced by PICKERING and Miss CANNON and embraces a great many types, of which we here describe the principal forms as they are defined in Part. II of Vol. XXVIII of the Annals of the Harvard Observatory. It may be remarked that PICKERING first arranged the types in alphabetical order A, B, C, &c., supposing, that order to correspond to the temperature of the stars. Later this was found to be partly wrong, and inparticular it was found that the B-stars may be hotter than those of type A. The following is the temperature-order of the spectra according to the opinion of the Harvard astronomers.

Type O (WOLF-RAYET stars). The spectra of these stars consist mainly of bright lines. They are characterized by the bright bands at wave-lengths $463\ \mu\mu$ and $469\ \mu\mu$, and the line at $501\ \mu\mu$ characteristic of gaseous nebulae is sometimes present.

This type embraces mainly stars of relatively small apparent brightness. The brightest is γ Velorum with $m = 2.22$. We shall find that the absolute magnitude of these stars nearly coincides with that of the stars of type B.

The type is grouped into five subdivisions represented by the letters Oa, Ob, Oc, Od and Oe. These subdivisions are conditioned by the varying intensities of the bright bands named above. The due sequence of these sub-types is for the present an open question.

Among interesting stars of this type is ζ Puppis (Od), in the spectrum of which PICKERING discovered a previously unknown series of helium lines. They were at first attributed (by RYDBERG) to hydrogen and were called "additional lines of hydrogen".

Type B (Orion type, Helium stars). All lines are here dark. Besides the hydrogen series we here find the He-lines (396, 403, 412, 414, 447, 471, 493 $\mu\mu$).

To this type belong all the bright stars (β, γ, δ, ε, ζ, η and others) in Orion with the exception of Betelgeuze. Further, Spica and many other bright stars.

On plate III ε Orionis is taken as representative of this type.

Type A (Sirius type) is characterized by the great intensity of the hydrogen lines (compare plate III). The helium lines have vanished. Other lines visible but faintly.

The greater part of the stars visible to the naked eye are found here. There are 1251 stars brighter than the 6th magnitude which belong to this type. Sirius, Vega, Castor, Altair, Deneb and others are all A-stars.

Type F (Calcium type). The hydrogen lines still rather prominent but not so broad as in the preceding type. The two calcium lines H and K (396.9, 393.4 mm) strongly pronounced.

Among the stars of this type are found a great many bright stars (compare the third chapter), such as Polaris, Canopus, Procyon.

Type G (Sun type). Numerous metallic lines together with relatively faint hydrogen lines.

To this class belong the sun, Capella, α Centauri and other bright stars.

Type K. The hydrogen lines still fainter. The K-line attains its maximum intensity (is not especially pronounced in the figure of plate III).

This is, next to the A-type, the most numerous type (1142 stars) among the bright stars.

We find here γ Andromedæ, β Aquilæ, Arcturus, α Cassiopeiæ, Pollux and Aldebaran, which last forms a transition to the next type.

Type M. The spectrum is banded and belongs to SECCHI's third type. The flutings are due to titanium oxide.

Only 190 of the stars visible to the naked eye belong to this type. Generally they are rather faint, but we here find Betelgeuze, α Herculis, β Pegasi, α Scorpii (Antares) and most variables of long period, which form a special sub-type *Md*, characterized by bright hydrogen lines together with the flutings.

Type *M* has two other sub-types Ma and Mb.

Type N (SECCHI's fourth type). Banded spectra. The flutings are due to compounds of carbon.

Here are found only faint stars. The total number is 241. All are red. 27 stars having this spectrum are variables of long period of the same type as Md.

The spectral types may be summed up in the following way:—

White	stars:—	SECCHI's type	I:—	Harvard	B and A,
Yellow	» :—	»	» II:—	»	F, G and K,
Red	» :—	»	» III:—	»	M,
»	» :—	»	» IV:—	»	N.

The Harvard astronomers do not confine themselves to the types mentioned above, but fill up the intervals between the types with sub-types which are designated by the name of the type followed by a numeral 0, 1, 2, . . ., 9. Thus the sub-types between A and F have the designations A0, A1, A2, . . ., A9, F0, &c. Exceptions are made as already indicated, for the extreme types O and M.

11. *Spectral index.* It may be gathered from the above description that the definition of the types implies many vague moments. Especially in regard to the G-type are very different definitions indeed accepted, even at Harvard.[1]) It is also a defect that the definitions do not directly give *quantitative* characteristics of the spectra. None the less it is possible to substitute for the spectral classes a continuous scale expressing the spectral character of a star. Such a scale is indeed implicit in the Harvard classification of the spectra.

Let us use the term *spectral index* (*s*) to define a number expressing the spectral character of a star. Then we may conveniently define this conception in the following way. Let A0 correspond to the spectral index $s = 0.0$, F0 to $s = +1.0$, G0 to $s = +2.0$, K0 to $s = +3.0$ M0 to $s = +4.0$ and B0 to $s = -1.0$. Further, let A1, A2, A3, &c., have

[1]) Compare H. A. 50 and H. A. 56 and the remarks in L. M. II, 19.

the spectral indices + 0.1, + 0.2, + 0.3, &c., and in like manner with the other intermediate sub-classes. Then it is evident that to all spectral classes between B0 and M there corresponds a certain spectral index *s*. The extreme types O and N are not here included. Their spectral indices may however be determined, as will be seen later.

Though the spectral indices, defined in this manner, are directly known for every spectral type, it is nevertheless not obvious that the series of spectral indices corresponds to a continuous series of values of some attribute of the stars. This may be seen to be possible from a comparison with another attribute which may be rather markedly graduated, namely the colour of the stars. We shall discuss this point in another paragraph. To obtain a well graduated scale of the spectra it will finally be necessary to change to some extent the definitions of the spectral types, a change which, however, has not yet been accomplished.

12. We have found in § 9 that the light-radiation of a star is described by means of the total intensity (I), the mean wave-length (λ_0) and the dispersion of the wave-length (σ_λ). λ_0 and σ_λ may be deduced from the spectral observations. It must here be observed that the observations give, not the intensities at different wave-lengths but, the values of these intensities as they are apprehended by the instruments employed—the eye or the photographic plate. For the derivation of the true curve of intensity we must know the distributive function of the instrument (L. M. 67). As to the eye, we have reason to believe, from the bolometric observations of LANGLEY (1888), that the mean wave-length of the visual curve of intensity nearly coincides with that of the true intensity-curve, a conclusion easily understood from DARWIN's principles of evolution, which demand that the human eye in the course of time shall be developed in such a way that the mean wave-length of the visual intensity curve does coincide with that of the true curve ($\lambda = 530\ \mu\mu$), when the greatest visual energy is obtained (L. M. 67). As to the dispersion, this is always greater in the true intensity-curve than in the visual curve, for which, according to § 10, it amounts to approximately 60 $\mu\mu$. We found indeed that the visual intensity curve is extended, approximately, from 400 $\mu\mu$ to 760 $\mu\mu$, a sixth part of which interval, approximately, corresponds to the dispersion σ of the visual curve.

In the case of the photographic intensity-curve the circumstances are different. The mean wave-length of the photographic curve is, approximately, 450 $\mu\mu$, with a dispersion of 16 $\mu\mu$, which is considerably smaller than in the visual curve.

13. Both the visual and the photographic curves of intensity differ according to the temperature of the radiating body and are therefore different for stars of different spectral types. Here the mean wave-length follows the formula of WIEN, which says that this wave-length varies inversely as the temperature. The total intensity, according to the law of STEPHAN, varies directly as the fourth power of the temperature. Even the dispersion is dependent on the variation of the temperature—directly as the mean wave-length, inversely as the temperature of the star (L. M. 41)—so that the mean wave-length, as well as the dispersion of the wave-length, is smaller for the hot stars O and B than for the cooler ones (K and M types). It is in this manner possible to determine the temperature of a star from a determination of its mean wave-length (λ_0) or from the dispersion in λ. Such determinations (from λ_0) have been made by SCHEINER and WILSING in Potsdam, by ROSENBERG and others, though these researches still have to be developed to a greater degree of accuracy.

14. *Effective wave-length.* The mean wave-length of a spectrum, or, as it is often called by the astronomers, the *effective* wave-length, is generally determined in the following way. On account of the refraction in the air the image of a star is, without the use of a spectroscope, really a spectrum. After some time of exposure we get a somewhat round image, the position of which is determined precisely by the mean wave-length. This method is especially used with a so-called *objective-grating,* which consists of a series of metallic threads, stretched parallel to each other at equal intervals. On account of the diffraction of the light we now get in the focal plane of the objective, with the use of these gratings, not only a fainter image of the star at the place where it would have arisen without grating, but also at both sides of this image secondary images, the distances of which from the central star are certain theoretically known multiples of the effective wave-lengths. In this simple manner it is possible to determine the effective wave-length, and this being a tolerably well-known function of the spectral-index, the latter can also be found. This method was first proposed by HERTZSPRUNG and has been extensively used by BERGSTRAND, LUNDMARK and LINDBLAD at the observatory of Upsala and by others.

15. *Colour-index.* We have already pointed out in § 9 that the colour may be identified with the mean wave-length (λ_0). As further λ_0 is closely connected with the spectral index (s), we may use the spectral index to represent the colour. Instead of s there may also be used another expression for the colour, called the colour-index. This expression was first introduced by SCHWARZSCHILD, and is defined in the following way.

We have seen that the zero-point of the photographic scale is chosen in such a manner that the visual magnitude m and the photographic magnitude m' coincide for stars of spectral index 0.0 (A0). The photographic magnitudes are then unequivocally determined. It is found that their values systematically differ from the visual magnitudes, so that for type B (and O) the photographic magnitudes are smaller than the visual, and the contrary for the other types. The difference is greatest for the M-type (still greater for the N-stars, though here for the present only a few determinations are known), for which stars if amounts to nearly two magnitudes. So much fainter is a red star on a photographic plate than when observed with the eye.

The difference between the photographic and the visual magnitudes is called the colour-index (c). The correlation between this index and the spectral-index is found to be rather high ($r = +0.96$). In L. M. II, 19 I have deduced the following tables giving the spectral-type corresponding to a given colour-index, and inversely.

TABLE I.

GIVING THE MEAN COLOUR-INDEX CORRESPONDING TO A GIVEN SPECTRAL TYPE OR SPECTRAL INDEX.

Spectral type	Spectral index	Colour-index
B0	− 1.0	− 0.46
B5	− 0.5	− 0.23
A0	0.0	0.00
A5	+ 0.5	+ 0.23
F0	+ 1.0	+ 0.46
F5	+ 1.5	+ 0.69

Spectral type	Spectral index	Colour-index
G0	+ 2.0	+ 0.92
G5	+ 2.5	+ 1.15
K0	+ 3.0	+ 1.38
K5	+ 3.5	+ 1.61
M0	+ 4.0	+ 1.84

TABLE 1*.

GIVING THE MEAN SPECTRAL INDEX CORRESPONDING TO A GIVEN COLOUR-INDEX.

Colour-index	Spectral index	Spectral type	Colour-index	Spectral index	Spectral type
−0.4	−0.70	B3	+1.0	+2.10	G1
−0.2	−0.30	B7	+1.2	+2.50	G5
0.0	+0.10	A1	+1.4	+2.90	G9
+0.2	+0.50	A5	+1.6	+3.30	K3
+0.4	+0.90	A9	+1.8	+3.70	K7
+0.6	+1.30	F3	+2.0	+4.10	M1
+0.8	+1.70	F7			

From each catalogue of visual magnitudes of the stars we may obtain their photographic magnitude through adding the colour-index. This may be considered as known (taking into account the high coefficient of correlation between s and c) as soon as we know the spectral type of the star. We may conclude directly that the number of stars having a photographic magnitude brighter than 6.0 is considerably smaller than the number of stars visually brighter than this magnitude. There are, indeed, 4701 stars for which $m < 6.0$ and 2874 stars having $m' < 6.0$.

16. *Radial velocity of the stars.* From the values of α and δ at different times we obtain the components of the proper motions of the stars perpendicular to the line of sight. The third component (W), in the radial direction, is found by the DOPPLER principle, through measuring the displacement of the lines in the spectrum, this displacement being towards the red or the violet according as the star is receding from or approaching the observer.

The velocity W will be expressed in siriometers per stellar year (sir./st.) and alternately also in km./sec. The rate of conversion of these units is given in § 5.

17. Summing up the remarks here given on the apparent attributes of the stars we find them referred to the following principal groups:—

I. *The position of the stars* is here generally given in galactic longitude (l) and latitude (b). Moreover their equatorial coordinates (α and δ) are given in an abridged notation ($\alpha\delta$), where the first four numbers give the right ascension in hours and minutes and the last two numbers give the declination in degrees, the latter being printed in italics if the declination is negative.

Eventually the position is given in galactic squares, as defined in § 2.

II. *The apparent motion of the stars* will be given in radial components (W) expressed in sir./st. and their motion perpendicular to the line of sight. These components will be expressed in one component (u_0) parallel to the galactic plane, and one component (v_0) perpendicular to it. If the distance (r) is known we are able to convert these components into components of the linear velocity perpendicular to the line of sight (U and V).

III. *The intensity of the light* of the stars is expressed in magnitudes. We may distinguish between the *apparent* magnitude (m) and the *absolute* magnitude (M), the latter being equal to the value of the apparent magnitude supposing the star to be situated at a distance of one siriometer.

The apparent magnitude may be either the *photographic* magnitude (m'), obtained from a photographic plate, or the *visual* magnitude (m) obtained with the eye.

The difference between these magnitudes is called the *colour-index* ($c = m' - m$).

IV. *The characteristics of the stellar radiation* are the mean wavelength (λ_0) and the dispersion (σ) in the wave-length. *The mean wavelength* may be either directly determined (perhaps as *effective* wavelength) or found from the spectral type (spectral index) or from the colour-index.

There are in all eight attributes of the stars which may be found from the observations:—the sphaerical position of the star (l, b), its distance (r), proper motion (u_0 and v_0), radial velocity (W), apparent magnitude (m or m'), absolute magnitude (M), spectral type (Sp) or spectral index (s), and colour-index (c). Of these the colour-index, the spectral type, the absolute magnitude and also (to a certain degree) the radial velocity may be considered as independent of the place of the observer and may therefore be considered not as only apparent but also as *absolute* attributes of the stars.

Between three of these attributes (m, M and r) a mathematical relation exists so that one of them is known as soon as the other two have been found from observations.

CHAPTER II.

SOURCES OF OUR PRESENT KNOWLEDGE OF THE STARS.

18. In this chapter I shall give a short account of the publications in which the most complete information on the attributes of the stars may be obtained, with short notices of the contents and genesis of these publications. It is, however, not my intention to give a history of these researches. We shall consider more particularly the questions relating to the position of the stars, their motion, magnitude, and spectra.

19. *Place of the stars. Durchmusterungs.* The most complete data on the position of the stars are obtained from the star catalogues known as "Durchmusterungs". There are two such catalogues, which together cover the whole sky, one—visual—performed in Bonn and called the *Bonner Durchmusterung* (B. D.), the other—photographic—performed in Cape *The Cape Photographic Durchmusterung* (C. P. D.). As the first of these catalogues has long been—and is to some extent even now—our principal scource for the study of the sky and is moreover the first enterprise of this kind, I shall give a somewhat detailed account of its origin and contents, as related by ARGELANDER in the introduction to the B. D.

B. D. was planned and performed by the Swedish-Finn ARGELANDER (born in Memel 1799). A scholar of BESSEL he was first called as director in Åbo, then in Hälsingfors, and from there went in 1836 to Bonn, where in the years 1852 to 1856 he performed this great *Durchmusterung.* As instrument he used a FRAUENHOFER comet-seeker with an aperture of 76 mm, a focal length of 650 mm, and 10 times magnifying power. The field of sight had an extension of 6°.

In the focus of the objective was a semicircular piece of thin glass, with the edge (= the diameter of the semicircle) parallel to the circle of declination. This edge was sharply ground, so that it formed a narrow dark line perceptible at star illumination. Perpendicular to this diameter (the "hour-line") were 10 lines, at each side of a middle line, drawn at a distance of 7′. These lines were drawn with black oil colour on the glass.

The observations are performed by the observer A and his assistant B. A is in a dark room, lies on a chair having the eye at the ocular and can easily look over 2° in declination. The assistant sits in the room below, separated by a board floor, at the *Thiede* clock.

From the beginning of the observations the declination circle is fixed at a certain declination (whole degrees). All stars passing the field at a distance smaller than one degree from the middle line are observed. Hence the name "Durchmusterung". When a star passes the "hour line" the magnitude is called out by A, and noted by B together with the time of the clock. Simultaneously the declination is noted by A in the darkness. On some occasions 30 stars may be observed in a minute.

The first observation was made on Febr. 25, 1852, the last on March 27, 1859. In all there were 625 observation nights with 1841 "zones". The total number of stars was 324 198.

The catalogue was published by ARGELANDER in three parts in the years 1859, 1861 and 1862[1]) and embraces all stars between the pole and 2° south of the equator brighter than $9^{m}.5$, according to the scale of ARGELANDER (his aim was to register all stars up to the 9th magnitude). To this scale we return later. The catalogue is arranged in accordance with the declination-degrees, and for each degree according to the right ascension. Quotations from B. D. have the form B. D. $23^{\circ}.174$, which signifies: Zone $+23^{\circ}$, star No. 174.

ARGELANDER's work was continued for stars between $\delta = -2^{\circ}$ and $\delta = -23^{\circ}$ by SCHÖNFELD, according to much the same plan, but with a larger instrument (aperture 159 mm, focal length 1930 mm, magnifying power 26 times). The observations were made in the years 1876 to 1881 and include 133 659 stars.[2])

The positions in B. D. are given in tenths of a second in right ascension and in tenths of a minute in declination.

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[1]) "Bonner Sternverzeichnis" in den Astronomischen Beobachtungen auf der Sternwarte zu Bonn, Dritter bis Fünfter Band. Bonn 1859—62.

[2]) "Bonner Durchmusterung", Vierte Sektion. Achter Band der Astronomischen Beobachtungen zu Bonn, 1886.

20. *The Cape Photographic Durchmusterung*[1]) (C. P. D.). This embraces the whole southern sky from —18° to the south pole. Planned by GILL, the photographs were taken at the Cape Observatory with a DALLMEYER lense with 15 cm aperture and a focal-length of 135 cm. Plates of 30 × 30 cm give the coordinates for a surface of 5 × 5 square degrees. The photographs were taken in the years 1885 to 1890. The measurements of the plates were made by KAPTEYN in Groningen with a "parallactic" measuring-apparatus specially constructed for this purpose, which permits of the direct obtaining of the right ascension and the declination of the stars. The measurements were made in the years 1886 to 1898. The catalogue was published in three parts in the years 1896 to 1900.

The positions have the same accuracy as in B. D. The whole number of stars is 454875. KAPTEYN considers the catalogue complete to at least the magnitude $9^m_.2$.

In the two great catalogues B. D. and C. P. D. we have all stars registered down to the magnitude 9.0 (visually) and a good way below this limit. Probably as far as to 10^m.

A third great Durchmusterung has for some time been in preparation at Cordoba in Argentina.[2]) It continues the southern zones of SCHÖNFELD and is for the present completed up to 62° southern declination.

All these Durchmusterungs are ultimately based on star catalogues of smaller extent and of great precision. Of these catalogues we shall not here speak (Compare, however, § 23).

A great "Durchmusterung", that will include all stars to the 11th magnitude, has for the last thirty years been in progress at different observatories proposed by the congress in Paris, 1888. The observations proceed very irregularly, and there is little prospect of getting the work finished in an appreciable time.

21. *Star charts.* For the present we possess two great photographic star charts, embracing the whole heaven:—The *Harvard Map* (H. M.) and the *FRANKLIN-ADAMS Charts* (F. A. C.).

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[1]) "The Cape Photographic Durchmusterung" by DAVID GILL and J. C. KAPTEYN, Annals of the Cape Observatory, vol. III—V (1896—1900).

[2]) "Cordoba Durchmusterung" by J. THOME. Results of the National Argentine Observatory, vol. 16, 17, 18, 21 (1894—1914).

The Harvard Map, of which a copy (or more correctly two copies) on glass has kindly been placed at the disposal of the Lund Observatory by Mr. PICKERING, embraces all stars down to the 11th magnitude. It consists of 55 plates, each embracing more than 900 square degrees of the sky. The photographs were taken with a small lens of only 2.5 cms. aperture and about 32.5 cms. focal-length. The time of exposure was one hour. These plates have been counted at the Lund Observatory by Hans HENIE. We return later to these counts.

The *FRANKLIN-ADAMS Charts* were made by an amateur astronomer FRANKLIN-ADAMS, partly at his own observatory (Mervel Hill) in England, partly in Cape and Johannesburg, Transvaal, in the years 1905—1912. The photographs were taken with a *Taylor* lens with 25 cm. aperture and a focal-length of 114 cm., which gives rather good images on a field of 15 × 15 square degrees.

The whole sky is here reproduced on, in all, 206 plates. Each plate was exposed for 2 hours and 20 minutes and gives images of the stars down to the 17th magnitude. The original plates are now at the observatory in Greenwich. Some copies on paper have been made, of which the Lund Observatory possesses one. It shows stars down to the 14th—15th magnitudes and gives a splendid survey of the whole sky more complete, indeed, than can be obtained, even for the north sky, by direct observation of the heavens with any telescope at present accessible in Sweden.

The F. A. C. have been counted by the astronomers of the Lund Observatory, so that thus a complete count of the number of stars for the whole heaven down to the 14th magnitude has been obtained. We shall later have an opportunity of discussing the results of these counts.

A great star map is planned in connection with the Paris catalogue mentioned in the preceding paragraph. This *Carte du Ciel* (C. d. C.) is still unfinished, but there seems to be a possibility that we shall one day see this work carried to completion. It will embrace stars down to the 14th magnitude and thus does not reach so far as the F. A. C., but on the other hand is carried out on a considerably greater scale and gives better images than F. A. C. and will therefore be of a great value in the future, especially for the study of the proper motions of the stars.

22. *Distance of the stars.* As the determination, from the annual parallax, of the distances of the stars is very precarious if the distance exceeds 5 sir. ($\pi = 0''.04$), it is only natural that the catalogues of star-distances should be but few in number. The most complete catalogues are those of BIGOURDAN in the Bulletin astronomique XXVI (1909), of KAPTEYN and WEERSMA in the publications of Groningen Nr. 24 (1910), embracing 365 stars, and of WALKEY in the "Journal of the British Astronomical Association XXVII" (1917), embracing 625 stars. Through the spectroscopie method of ADAMS it will be possible to enlarge this number considerably, so that the distance of all stars, for which the spectrum is well known, may be determined with fair accuracy. ADAMS has up to now published 1646 parallax stars.

23. *Proper motions.* An excellent catalogue of the proper motions of the stars is LEWIS BOSS'S "Preliminary General Catalogue of 6188 stars" (1910) (B. P. C.). It contains the proper motions of all stars down to the sixth magnitude (with few exceptions) and moreover some fainter stars. The catalogue is considered by the editor only as a preliminary to a greater catalogue, which is to embrace some 25000 stars and is now nearly completed.

24. *Visual magnitudes.* The Harvard observatory has, under the direction of PICKERING, made to its principal aim to study the magnitudes of the stars, and the history of this observatory is at the same time the history of the treatment of this problem. PICKERING, in the genuine american manner, is not satisfied with the three thirds of the sky visible from the Harvard observatory, but has also founded a daughter observatory in South America, at Arequipa in Peru. It is therefore possible for him to publish catalogues embracing the whole heaven from pole to pole. The last complete catalogue (1908) of the magnitudes of the stars is found in the "Annals of the Harvard Observatory T. 50" (H. 50). It contains 9110 stars and can be considered as complete to the magnitude $6^m.5$. To this catalogue are generally referred the magnitudes which have been adopted at the Observatory of Lund, and which are treated in these lectures.

A very important, and in one respect even still more comprehensive, catalogue of visual magnitudes is the "Potsdam General Catalogue" (P. G. C.) by MÜLLER and KEMPF, which was published simultaneously with H. 50. It contains the magnitude of 14 199 stars and embraces all stars on the northern hemisphere brighter than $7^m.5$ (according to B. D.). We have already seen that the zero-point of H. 50 and P. G. C. is somewhat different and that the magnitudes in P. G. C. must be increased by $-0^m.16$ if they are to be reduced to the Harvard scale. The difference between the two catalogues however is due to some extent to the colour of the stars, as has been shown by Messrs. MÜLLER and KEMPF.

25. *Photographic magnitudes.* Our knowledge of this subject is still rather incomplete. The most comprehensive catalogue is the "Actinometrie" by SCHWARZSCHILD (1912), containing the photographic magnitudes of all stars in B. D. down to the magnitude $7^m.5$ between the equator and a declination of $+20°$. In all, 3522 stars. The photographic magnitudes are however not reduced for the zero-point (compare § 6).

These is also a photometric photographic catalogue of the stars nearest to the pole in PARKHURST's "Yerkes actinometry" (1912),[1]) which contains all stars in B. D. brighter than $7^m.5$ between the pole and 73° northern declination. The total number of stars is 672.

During the last few years the astronomers of Harvard and Mount Wilson have produced a collection of "standard photographic magnitudes" for faint stars. These stars, which are called the *polar sequence*,[2]) all lie in the immediate neighbourhood of the pole. The list is extended down to the 20th magnitude. Moreover similar standard photographic magnitudes are given in H. A. 71, 85 and 101.

A discussion of the *colour-index* (*i. e.*, the difference between the photographic and the visual magnitudes) will be found in L. M. II, 19. When the visual magnitude and the type of spectrum are known, the photographic magnitude may be obtained, with a generally sufficient accuracy, by adding the colour-index according to the table 1 in § 15 above.

26. *Stellar spectra.* Here too we find the Harvard Observatory to be the leading one. The same volume of the Annals of the Harvard Observatory (H. 50) that contains the most complete catalogue of visual magnitudes, also gives the spectral types for all the stars there included, *i. e.*, for all stars to $6^m.5$. Miss CANNON, at the Harvard Observatory, deserves the principal credit for this great work. Not content with

[1]) Aph. J., vol. 36.

[2]) H. A., vol. 71.

this result she is now publishing a still greater work embracing more than 200000 stars. The first four volumes of this work are now published and contain the first twelve hours of right ascension, so that half the work is now printed.[1])

27. *Radial velocity.* In this matter, again, we find America to be the leading nation, though, this time, it is not the Harvard or the Mount Wilson but the Lick Observatory to which we have to give the honour. The eminent director of this observatory, W. W. CAMPBELL, has in a high degree developed the accuracy in the determination of radial velocities and has moreover carried out such determinations in a large scale. The "Bulletin" No. 229 (1913) of the Lick Observatory contains the radial velocity of 915 stars. At the observatory of Lund, where as far as possible card catalogues of the attributes of the stars are collected, GYLLENBERG has made a catalogue of this kind for the radial velocities. The total number of stars in this catalogue now amounts to 1640.[2])

28. Finally I shall briefly mention some comprehensive works on more special questions regarding the stellar system.

On *variable stars* there is published every year by HARTWIG in the "Vierteljahrschrift der astronomischen Gesellschaft" a catalogue of all known variable stars with needful information about their minima &c. This is the completest and most reliable of such catalogues, and is always up to date.

On *nebulae* we have the excellent catalogues of DREYER, the "New General Catalogue" (N. G. C.) of 1890 in the "Memoirs of the Astronomical Society" vol. 49, the "Index catalogues" (I. C.) in the same memoirs, vols. 51 and 59 (1895 and 1908). These catalogues contain all together 13226 objects.

Regarding other special attributes I refer in the first place to the important Annals of the Harvard Observatory. Other references will be given in the following, as need arises.

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[1]) H. A., vol. 91, 92, 93, 94.

[2]) A catalogue of radial velocities has this year been published by J. VOUTE, embracing 2071 stars. "First catalogue of radial velocities", by J. VOUTE. Weltevreden, 1920.

CHAPTER III.

SOME GROUPS OF KNOWN STARS.

29. The number of cases in which all the eight attributes of the stars discussed in the first chapter are well known for one star is very small, and certainly does not exceed one hundred. These cases refer principally to such stars as are characterized either by great brilliancy or by a great proper motion. The principal reason why these stars are better known than others is that they lie rather near our solar system. Before passing on to consider the stars from more general statistical points of view, it may therefore be of interest first to make ourselves familiar with these well-known stars, strongly emphasizing, however, the exceptional character of these stars, and carefully avoiding any generalization from the attributes we shall here find.

30. *The apparently brightest stars.* We begin with these objects so well known to every lover of the stellar sky. The following table contains all stars the apparent visual magnitude of which is brighter than $1^{m}.5$.

The first column gives the current number, the second the name, the third the equatorial designation ($\alpha\,\delta$). It should be remembered that the first four figures give the hour and minutes in right ascension, the last two the declination, italics showing negative declination. The fourth column gives the galactic square, the fifth and sixth columns the galactic longitude and latitude. The seventh and eighth columns give the annual parallax and the corresponding distance expressed in siriometers. The ninth column gives the proper motion (μ), the tenth the radial velocity W expressed in sir./st. (To get km./sec. we may multiply by 4.7375). The eleventh column gives the apparent visual magnitude, the twelfth column the absolute magnitude (M), computed from m with the help of r. The 13th column gives the type of spectrum (Sp), and the last column the photographic magnitude (m'). The difference between m' and m gives the colour-index (c).

TABLE 2.

THE APPARENTLY BRIGHTEST STARS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Name	Position				Distance		Motion		Magnitude		Spectrum	
		($\alpha\delta$)	Square	l	b	π	r	μ	W	m	M	Sp	m'
							sir.		sir./st.				m'
1	Sirius	(064016)	GD_7	195°	−8°	0″.376	0.5	1″.82	−1.56	$-1^m.58$	$-0^m.3$	A	−1.58
2	Canopus	(062152)	GD_8	229	−24	0.007	29.5	0.02	+4.39	−0.86	−8.2	F	−0.40
3	Vega	(183338)	GC_2	30	+17	0.094	2.2	0.35	−2.91	0.14	−1.6	A	0.14
4	Capella	(050945)	GC_5	131	+5	0.066	3.1	0.44	+6.38	0.21	−2.3	G	1.18
5	Arcturus	(141119)	GA_2	344	+68	0.075	2.7	2.28	−0.82	0.24	−1.9	K	1.62
6	α Centauri	(143260)	GD_{10}	284	−2	0.759	0.3	3.68	−4.69	0.33	+3.2	G	1.25
7	Rigel	(050908)	GD_6	176	−24	0.007	29.5	0.00	+4.77	0.34	−7.0	B8p	0.25
8	Procyon	(073405)	GC_7	182	+14	0.324	0.6	1.24	−0.74	0.48	+1.5	F5	1.17
9	Achernar	(013457)	GE_8	256	−59	0.051	4.0	0.09	..	0.60	−2.4	B5	0.37
10	β Centauri	(135659)	GC_{10}	280	+2	0.037	5.6	0.04	+2.53	0.86	−2.9	B1	0.45
11	Altair	(194508)	GD_1	15	−10	0.238	0.9	0.66	−6.97	0.89	+1.2	A5	1.12
12	Betelgeuze	(054907)	GD_6	168	−8	0.030	6.9	0.08	+4.43	0.92	−3.3	Ma	2.76
13	Aldebaran	(043016)	GD_5	149	−19	0.073	2.8	0.20	+11.63	1.06	−1.2	K5	2.67
14	Pollux	(073928)	GC_6	160	+25	0.064	3.2	0.07	+0.82	1.21	−1.3	K	2.59
15	Spica	(131910)	GB_8	286	+51	..	..	0.06	+0.34	1.21	..	B2	0.84
16	Antares	(162326)	GC_{11}	320	+14	0.029	7.1	0.03	−0.63	1.22	−3.0	Map	3.06
17	Fomalhaut	(225230)	GE_{10}	348	−66	0.138	1.5	0.37	+1.41	1.29	+0.4	A3	1.48
18	Deneb	(203844)	GC_2	51	+1	..	..	0.00	−0.84	1.33	..	A2	1.42
19	Regulus	(100312)	GB_6	196	+50	0.033	6.3	0.25	..	1.34	−2.7	B8	1.25
20	β Crucis	(124159)	GC_{10}	270	+3	0.008	25.8	0.06	+2.74	1.50	−5.6	B1	1.09
							sir.						m'
	Mean...	..	..	..	23°.5	0″.134	7.3	0″.56	3.26	$+0^m.64$	$-2^m.1$	F1	+1.13

The values of $(\alpha\delta)$, m, Sp are taken from H. 50. The values of l, b are computed from $(\alpha\delta)$ with the help of tables in preparation at the Lund Observatory, or from the original to plate I at the end, allowing the conversion of the equatorial coordinates into galactic ones. The values of π are generally taken from the table of KAPTEYN and WEERSMA mentioned in the previous chapter. The values of μ are obtained from B. P. C., those of the radial velocity (W) from the card catalogue in Lund already described.

There are in all, in the sky, 20 stars having an apparent magnitude brighter than $1^{m}.5$. The brightest of them is *Sirius*, which, owing to its brilliancy and position, is visible to the whole civilized world. It has a spectrum of the type A0 and hence a colour-index nearly equal to 0.0 (observations in Harvard give $c = +0.06$). Its apparent magnitude is $-1^{m}.6$, nearly the same as that of Mars in his opposition. Its absolute magnitude is $-0^{m}.3$, *i. e.*, fainter than the apparent magnitude, from which we may conclude that it has a distance from us smaller than one siriometer. We find, indeed, from the eighth column that $r = 0.5$ sir. The proper motion of Sirius is $1''.32$ per year, which is rather large but still not among the largest proper motions as will be seen below. From the 11th column we find that Sirius is moving towards us with a velocity of 1.6 sir./st. (= 7.6 km./sec.), a rather small velocity. The third column shows that its right ascension is $6^{h}\ 40^{m}$ and its declination $-16°$. It lies in the square GD_7 and its galactic coordinates are seen in the 5th and 6th columns.

The next brightest star is *Canopus* or α Carinæ at the south sky. If we might place absolute confidence in the value of M ($= -8.2$) in the 12th column this star would be, in reality, a much more imposing apparition than Sirius itself. Remembering that the apparent magnitude of the moon, according to § 6, amounts to -11.6, we should find that Canopus, if placed at a distance from us equal to that of Sirius ($r =$ 0.5 sir.), would shine with a lustre equal to no less than a quarter of that of the moon. It is not altogether astonishing that a fanciful astronomer should have thought Canopus to be actually the central star in the whole stellar system. We find, however, from column 8 that its supposed distance is not less that 30 sir. We have already pointed out that distances greater than 4 sir., when computed from annual parallaxes, must generally be considered as rather uncertain. As the value of M

is intimately dependent on that of *r* we must consider speculations based on this value to be very vague. Another reason for a doubt about a great value for the real luminosity of this star is found from its type of spectrum which, according to the last column, is F0, a type which, as will be seen, is seldom found among giant stars. A better support for a large distance could on the other hand be found from the small proper motion of this star. Sirius and Canopus are the only stars in the sky having a negative value of the apparent visual magnitude.

Space will not permit us to go through this list star for star. We may be satisfied with some general remarks.

In the fourth column is the galactic square. We call to mind that all these squares have the same area, and that there is therefore the same probability *a priori* of finding a star in one of the squares as in another. The squares GC and GD lie along the galactic equator (the Milky Way). We find now from column 4 that of the 20 stars here considered there are no less than 15 in the galactic equator squares and only 5 outside, instead of 10 in the galactic squares and 10 outside, as would have been expected. The number of objects is, indeed, too small to allow us to draw any cosmological conclusions from this distribution, but we shall find in the following many similar instances regarding objects that are principally accumulated along the Milky Way and are scanty at the galactic poles. We shall find that in these cases we may *generally* conclude from such a partition that we then have to do with objects *situated far from the sun,* while objects that are uniformly distributed on the sky lie relatively near us. It is easy to understand that this conclusion is a consequence of the supposition, confirmed by all star counts, that the stellar system extends much farther into space along the Milky Way than in the direction of its poles.

If we could permit ourselves to draw conclusions from the small material here under consideration, we should hence have reason to believe that the bright stars lie relatively far from us. In other words we should conclude that the bright stars seem to be bright to us not because of their proximity but because of their large intrinsic luminosity. Column 8 really tends in this direction. Certainly the distances are not in this case colossal, but they are nevertheless sufficient to show, in some degree, this uneven partition of the bright stars on the sky. The mean distance of these stars is as large as 7.5 sir. Only α Centauri,

3

Sirius, Procyon and Altair lie at a distance smaller than one siriometer. Of the other stars there are two that lie as far as 30 siriometers from our system. These are the two giants Canopus and Rigel. Even if, as has already been said, the distances of these stars may be considered as rather uncertain, we must regard them as being rather large.

As column 8 shows that these stars are rather far from us, so we find from column 12, that their absolute luminosity is rather large. The mean absolute magnitude is, indeed, $-2^{m}.1$. We shall find that only the greatest and most luminous stars in the stellar system have a negative value of the absolute magnitude.

The mean value of the proper motions of the bright stars amounts to $0''.56$ per year and may be considered as rather great. We shall, indeed, find that the mean proper motion of the stars down to the 6^{th} magnitude scarcely amounts to a tenth part of this value. On the other hand we find from the table that the high value of this mean is chiefly due to the influence of four of the stars which have a large proper motion, namely Sirius, Arcturus, α Centauri and Procyon. The other stars have a proper motion smaller than $1''$ per year and for half the number of stars the proper motion amounts to approximately $0''.05$, indicating their relatively great distance.

That the absolute velocity of these stars is, indeed, rather small may be found from column 10, giving their radial velocity, which in the mean amounts to only three siriometers per stellar year. From the discussion below of the radial velocities of the stars we shall find that this is a rather small figure. This fact is intimately bound up with the general law in statistical mechanics, to which we return later, that stars with large masses generally have a small velocity. We thus find in the radial velocities fresh evidence, independent of the distance, that these bright stars are giants among the stars in our stellar system.

We find all the principal spectral types represented among the bright stars. To the helium stars (B) belong Rigel, Achernar, β Centauri, Spica, Regulus and β Crucis. To the Sirius type (A) belong Sirius, Vega, Altair, Fomalhaut and Deneb. To the Calcium type (F) Canopus and Procyon. To the sun type (G) Capella and α Centauri. To the K-type belong Arcturus, Aldebaran and Pollux and to the M-type the two red stars Betelgeuze and Antares. Using the spectral indices as an expression for the spectral types we find that the mean spectral index of these stars is $+1.1$, corresponding to the spectral type F1.

31. *Stars with the greatest proper motion.* In table 3 I have collected the stars having a proper motion greater than 3″ per year. The designations are the same as in the preceding table, except that the names of the stars are here taken from different catalogues.

In the astronomical literature of the last century we find the star 1830 Groombridge designed as that which possesses the greatest known proper motion. It is now distanced by two other stars C. P. D. $5^{h}.243$ discovered in the year 1897 by KAPTEYN and INNES on the plates taken for the Cape Photographic Durchmusterung, and BARNARD's star in Ophiuchus, discovered 1916. The last-mentioned star, which possesses the greatest proper motion now known, is very faint, being only of the 10th magnitude, and lies at a distance of 0.40 sir. from our sun and is hence, as will be found from table 5 the third nearest star for which we know the distance. Its linear velocity is also very great, as we find from column 10, and amounts to 19 sir./st. (= 90 km./sec.) in the direction towards the sun. The absolute magnitude of this star is $11^{m}.7$ and it is, with the exception of one other, the very faintest star now known. Its spectral type is Mb, a fact worth fixing in our memory, as different reasons favour the belief that it is precisely the M-type that contains the very faintest stars. Its apparent velocity (*i. e.*, the proper motion) is so great that the star in 1000 years moves 3°, or as much as 6 times the diameter of the moon. For this star, as well as for its nearest neighbours in the table, observations differing only by a year are sufficient for an approximate determination of the value of the proper motion, for which in other cases many tenths of a year are required.

Regarding the distribution of these stars in the sky we find that, unlike the brightest stars, they are not concentrated along the Milky Way. On the contrary we find only 6 in the galactic equator squares and 12 in the other squares. We shall not build up any conclusion on this irregularity in the distribution, but supported by the general thesis of the preceding paragraph we conclude only that these stars must be relatively near us. This follows, indeed, directly from column 8, as not less than eleven of these stars lie within one siriometer from our sun. Their mean distance is 0.87 sir.

TABLE 3.

STARS WITH THE GREATEST PROPER MOTION.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Name	Position				Distance		Motion		Magnitude		Spectrum	
		$(\alpha\,\delta)$	Square	l	b	π	r	μ	W	m	M	Sp.	m'
							sir.		sir./st.				m'
1	Barnards star........	(175204)	GC_{12}	358°	+ 12°	0″.515	0.40	10″.29	— 19	$9^m.7$	+ $11^m.7$	M b	11.5
2	C. Z. $5^h.243$	(050744)	GE_7	218	— 35	0.319	0.65	8.75	+ 51	9.2	+ 10.1	K 2	10.6
3	Groom. 1830.........	(114738)	GA_1	135	+ 75	0.102	2.02	7.06	— 20	6.5	+ 5.0	G 5	7.6
4	Lac. 9352	(225936)	GE_{10}	333	— 66	0.292	0.71	6.90	+ 2	7.5	+ 8.2	K	8.9
5	C. G. A. 32416........	(235937)	GF_2	308	— 75	0.230	0.89	6.11	+ 5	8.2	+ 8.5	G	9.1
6	61 Cygni	(210238)	GD_2	50	— 7	0.311	0.66	5.27	— 13	5.6	+ 6.5	K 5	7.2
7	Lal. 21185	(105736)	GB_5	153	+ 66	0.403	0.51	4.77	— 18	7.6	+ 9.1	M b	8.9
8	ε Indi	(215557)	GE_9	304	— 47	0.284	0.73	4.70	— 8	4.7	+ 5.4	K 5	6.3
9	Lal. 21258	(110044)	GB_4	135	+ 64	0.203	1.02	4.47	+ 14	8.5	+ 8.5	M a	10.3
10	0^2 Eridani............	(041007)	GE_5	168	— 36	0.174	1.19	4.11	— 9	4.7	+ 4.3	G 5	5.8
11	Proxima Centauri....	(142262)	GD_{10}	281	— 2	0.780	0.26	3.85	..	11.0	+ 13.9	..	13.5
12	Oe. A. 14320	(150415)	GB_9	314	+ 35	0.035	5.90	3.75	+ 61	9.0	+ 5.1	G 0	9.9
13	μ Cassiopeiæ	(010154)	GD_4	93	— 8	0.112	1.84	3.73	— 21	5.7	+ 4.4	G 3	6.8
14	α Centauri..	(143260)	GD_{10}	284	— 2	0.759	0.27	3.68	— 5	0.3	+ 3.2	G	1.2
15	Lac. 8760	(211139)	GE_{10}	332	— 44	0.248	0.83	3.53	+ 3	6.6	+ 7.0	G	7.5
16	Lac. 1060	(031543)	GE_7	216	— 55	0.162	1.27	3.05	+ 18	5.6	+ 5.1	G 5	6.7
17	Oe. A. 11677	(111466)	GB_8	103	+ 50	0.198	1.04	3.03	..	9.2	+ 9.1	M a	11.0
18	Van Maanens star	(004304)	GD_8	92	— 58	0.246	0.84	3.01	..	12.3	+ 12.7	F 0	12.9
							sir.		sir./st.				m'
	Mean ...	.	..	..	41°	0″.298	0.87	5″.00	17.8	$7^m.3$	+ $7^m.6$	G8	8.7

That the great proper motion does not depend alone on the proximity of these stars is seen from column 10, giving the radial velocities. For some of the stars (4) the radial velocity is for the present unknown, but the others have, with few exceptions, a rather great velocity amounting in the mean to 18 sir./st. (= 85 km./sec.), if no regard is taken to the sign, a value nearly five times as great as the absolute velocity of the sun. As this is only the component along the line of sight, the absolute velocity is still greater, approximately equal to the component velocity multiplied by $\sqrt{2}$. We conclude that the great proper motions depend partly on the proximity, partly on the great linear velocities of the stars. That both these attributes here really cooperate may be seen from the absolute magnitudes (M).

The apparent and the absolute magnitudes are for these stars nearly equal, the means for both been approximately 7^m. This is a consequence of the fact that the mean distance of these stars is equal to one siriometer, at which distance m and M, indeed, do coincide. We find that these stars have a small luminosity and may be considered as *dwarf* stars. According to the general law of statistical mechanics already mentioned small bodies upon an average have a great absolute velocity, as we have, indeed, already found from the observed radial velocities of these stars.

As to the spectral type, the stars with great proper motions are all yellow or red stars. The mean spectral index is + 2.8, corresponding to the type G8. If the stars of different types are put together we get the table

Type	*Number*	*Mean value of M*
G	8	5.3
K	4	7.5
M	4	9.6

We conclude that, at least for these stars, the mean value of the absolute magnitude increases with the spectral index. This conclusion, however, is not generally valid.

32. *Stars with the greatest radial velocities.* There are some kinds of nebulae for which very large values of the radial velocities have been found. With these we shall not for the present deal, but shall confine ourselves to the stars. The greatest radial velocity hitherto found is possessed by the star (040822) of the eighth magnitude in the constellation Perseus, which retires from us with a velocity of 72 sir./st. or 341 km./sec. The nearest velocity is that of the star (010361) which approaches us with approximately the same velocity. The following table contains all stars with a radial velocity greater than 20 sir./st. (= 94.8 km./sec.). It is based on the catalogue of VOUTE mentioned above.

Regarding their distribution in the sky we find 11 in the galactic equator squares and 7 outside. A large radial velocity seems therefore to be a galactic phenomenon and to be correlated to a great distance from us. Of the 18 stars in consideration there is only one at a distance smaller than one siriometer and 2 at a distance smaller than 4 siriometers. Among the nearer ones we find the star (050744), identical with C. P. D. $5^{h}.243$, which was the "second" star with great proper motion. These stars have simultaneously the greatest proper motion and very great linear velocity. Generally we find from column 9 that these stars with large radial velocity possess also a large proper motion. The mean value of the proper motions amounts to $1''.34$, a very high value.

In the table we find no star with great apparent luminosity. The brightest is the 10th star in the table which has the magnitude 5.1. The mean apparent magnitude is 7.7. As to the absolute magnitude (M) we see that most of these speedy stars, as well as the stars with great proper motions in table 3, have a rather great *positive* magnitude and thus are absolutely faint stars, though they perhaps may not be directly considered as dwarf stars. Their mean absolute magnitude is + 3.0.

Regarding the spectrum we find that these stars generally belong to the yellow or red types (G, K, M), but there are 6 F-stars and, curiously enough, two A-stars. After the designation of their type (A 2 and A 3) is the letter *p* (= peculiar), indicating that the spectrum in some respect differs from the usual appearance of the spectrum of this type. In the present case the peculiarity consists in the fact that a line of the wave-length 448.1, which emanates from magnesium and which we may find on plate III in the spectrum of Sirius, does not occur in the spectrum of these stars, though the spectrum has otherwise the same appearence as in the case of the Sirius stars. There is reason to suppose that the absence of this line indicates a low power of radiation (low. temperature) in these stars (compare ADAMS).

TABLE 4.

STARS WITH THE GREATEST RADIAL VELOCITY.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Name	Position				Distance		Motion		Magnitude		Spectrum	
		($\alpha\,\delta$)	Square	l	b	π	r	μ	W	m	M	Sp	m'
1	A. G. Berlin 1366	(040822)	GD_5	141°	− 20°	0″.007	sir. 30.8	0″.54	sir./st. + 72	$8^m.9$	+ $1^m.4$	F0	m' 9.4
2	Lal. 1966	(010361)	GD_4	93	− 2	0.016	12.9	0.61	− 69	7.9	+ 2.3	F3	8.5
3	A. Oe. 14320	(150415)	GB_9	314	+ 35	0.035	5.9	3.75	+ 61	9.0	+ 5.1	G0	9.9
4	C. Z. $5^h.243$	(050744)	GE_7	218	− 35	0.319	0.6	8.75	+ 51	9.2	+ 10.1	K2	10.6
5	Lal. 15290	(074730)	GC_6	158	+ 26	0.023	9.0	1.96	− 51	8.2	+ 3.4	G0	9.1
6	53 Cassiop.	(015563)	GC_4	98	+ 2	. .	. .	0.01	− 44	5.6	. .	B8	5.5
7	A. G. Berlin 1866	(055719)	GD_6	159	− 2	0.021	9.8	0.76	− 40	9.0	+ 4.0	F0	9.9
8	W Lyræ	(181136)	GC_2	31	+ 21	. .	. .	. .	− 39	var.	. .	Md	var.
9	Boss 1511	(055926)	GD_7	200	− 20	0.012	17.0	0.10	+ 39	5.2	− 1.0	G5	6.1
10	ω Pavonis	(184960)	GD_{11}	304	− 24	. .	. .	0.14	+ 38	5.1	. .	K	6.5
11	A. Oe. 20452	(201721)	GE_{10}	351	− 31	0.015	13.5	1.18	− 38	8.1	+ 2.1	G8p	9.4
12	Lal. 28607	(153710)	GB_{10}	325	+ 34	0.033	6.2	1.18	− 36	7.3	+ 3.3	A2p	7.4
13	A. G. Leiden 5734	(161132)	GB_1	21	+ 45	0.002	89.2	0.04	− 35	8.3	− 1.5	K4	9.9
14	Lal. 37120	(192932)	GC_2	33	+ 6	0.050	4.1	0.52	− 34	6.6	+ 3.5	G2	7.6
15	Lal. 27274	(145421)	GB_9	308	+ 34	0.013	16.2	0.79	+ 34	8.3	+ 2.2	F4	8.9
16	Lal. 5761	(030225)	GD_5	126	− 28	0.039	5.1	0.86	− 32	8.0	+ 4.4	A3p	8.1
17	W. B. $17^h.517$	(172906)	GC_{12}	358	+ 20	0.014	14.1	0.63	− 31	8.6	+ 2.8	F1	9.1
18	Lal. 23995	(124717)	GB_8	271	+ 46	0.012	17.0	0.88	+ 30	8.2	+ 2.0	F3	8.8
	Mean	. .	. .	. .	23°.9	0″.041	sir. 16.7	1″.34	sir./st. 43.0	$7^m.7$	+ $3^m.0$	F9	m' 8.5

33. *The nearest stars.* The star α in Centaurus was long considered as the nearest of all stars. It has a parallax of $0''.75$, corresponding to a distance of 0.27 siriometers (= 4.26 light years). This distance is obtained from the annual parallax with great accuracy, and the result is moreover confirmed in another way (from the study of the orbit of the companion of α Centauri). In the year 1916 INNES discovered at the observatory of Johannesburg in the Transvaal a star of the 10th magnitude, which seems to follow α Centauri in its path in the heavens, and which, in any case, lies at the same distance from the earth, or somewhat nearer. It is not possible at present to decide with accuracy whether *Proxima Centauri*—as the star is called by INNES—or α Centauri is our nearest neighbour. Then comes BARNARD's star (175204), whose large proper motion we have already mentioned. As No. 5 we find Sirius, as No. 8 Procyon, as No. 21 Altair. The others are of the third magnitude or fainter. No. 10—61 Cygni—is especially interesting, being the first star for which the astronomers, after long and painful endeavours in vain, have succeeded in determining the distance with the help of the annual parallax (BESSEL 1841).

From column 4 we find that the distribution of these stars on the sky is tolerably uniform, as might have been predicted. All these stars have a large proper motion, this being in the mean $3''.42$ per year. This was a priori to be expected from their great proximity. The radial velocity is, numerically, greater than could have been supposed. This fact is probably associated with the generally small mass of these stars.

Their apparent magnitude is upon an average 6.3. The brightest of the near stars is Sirius ($m = -1.6$), the faintest Proxima Centauri ($m = 11$). Through the systematic researches of the astronomers we may be sure that no bright stars exist at a distance smaller than one siriometer, for which the distance is not already known and well determined. The following table contains without doubt—we may call them briefly all *near* stars—all stars within one siriometer from us with an apparent magnitude brighter than 6^m (the table has 8 such stars), and probably also all near stars brighter than 7^m (10 stars), or even all brighter than the eighth magnitude (the table has 13 such stars and two near the limit). Regarding the stars of the eighth magnitude or fainter no systematic investigations of the annual parallax have been made and among these stars we may get from time to time a new star belonging to the sirio-

TABLE 5.

THE NEAREST STARS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Name	Position				Distance		Motion		Magnitude		Spectrum	
		($\alpha\,\delta$)	Square	l	b	π	r	μ	W	m	M	Sp	m'
							sir.		sir./st.				m'
1	Proxima Centauri....	(1422*62*)	GD_{10}	281°	— 2°	0″.780	0.26	3″.85	..	11^m.0	+13^m.9	..	13.5
2	*α* Centauri...........	(1432*60*)	GD_{10}	284	— 2	0.759	0.27	3.68	— 5	0.33	+ 3.2	G	1.25
3	Barnards p. m. star...	(175204)	GC_{12}	358	+ 12	0.515	0.40	10.29	— 19	9.7	+11.7	Mb	11.5
4	Lal. 21 185...........	(105736)	GB_5	153	+ 66	0.403	0.51	4.77	— 18	7.6	+ 9.1	Mb	8.9
5	Sirius...............	(0640*16*)	GD_7	195	— 8	0.376	0.55	1.32	— 2	—1.58	— 0.3	A	—1.58
6	..	(111357)	GC_6	158	+ 3	0.337	0.60	2.72	..	..	..	..	12
7	*τ* Ceti...............	(0139*16*)	GF_1	144	— 74	0.334	0.62	1.92	— 3	3.6	+ 4.6	K0	4.6
8	Procyon..............	(073405)	GC_7	182	+ 14	0.324	0.64	1.24	— 1	0.48	+ 1.5	F5	0.90
9	C. Z. 5^h.243...........	(0507*44*)	GE_7	218	— 35	0.319	0.65	8.75	+ 51	9.2	+10.1	K2	10.6
10	61 Cygni.............	(210238)	GD_2	50	— 7	0.311	0.66	5.27	— 13	5.6	+ 6.5	K5	7.2
11	Lal. 26 481...........	(1425*15*)	GB_9	124	— 40	0.311	0.66	0.47	..	7.8	+ 8.7	G5	8.9
12	*ε* Eridani............	(0328*09*)	GE_5	153	— 42	0.295	0.70	0.97	+ 3	3.8	+ 4.6	K0	4.8
13	Lac. 9352............	(2259*36*)	GE_{10}	333	— 66	0.292	0.71	6.90	+ 2	7.5	+ 8.2	K	8.9
14	Pos. Med. 2164......	(184159)	GC_2	56	+ 24	0.292	0.71	2.28	..	8.9	+ 9.6	K	10.3
15	*ε* Indi..............	(2155*57*)	GE_9	304	— 47	0.284	0.73	4.70	— 8	4.7	+ 5.4	K5	6.3
16	Groom. 34...........	(001243)	GD_3	84	— 20	0.281	0.73	2.89	+ 1	8.1	+ 8.8	Ma	9.5
17	Oe. A. 17 415.........	(173768)	GC_3	65	+ 32	0.268	0.77	1.30	..	9.1	+ 9.7	K	10.5
18	Krüger 60............	(222457)	GC_3	72	0	0.256	0.81	0.94	..	9.2	+ 9.6	K5	10.8
19	Lac. 8760............	(2111*39*)	GE_{10}	332	— 44	0.248	0.88	3.53	+ 3	6.6	+ 7.0	G	7.5
20	van Maanens p. m. star	(004304)	GE_3	92	— 58	0.216	0.84	3.01	..	12.3	+12.7	F0	12.9
21	Altair...............	(194508)	GD_1	15	— 10	0.238	0.87	0.66	— 7	0.9	+ 1.2	A5	1.12
22	C. G. A. 32 416.......	(2359*37*)	GF_2	308	— 75	0.230	0.89	6.11	+ 5	8.2	+ 8.5	G	9.1
23	Bradley 1584.........	(1129*32*)	GC_6	252	+ 28	0.216	0.95	1.06	— 5	6.1	+ 6.2	G	6.9
							sir.		sir./st.				m'
	Mean...	..	..	..	30°.8	0″.344	0.67	3″.42	9.1	6^m.3	+ 7^m.3	G6	7.5

meter sphere in the neighbourhood of the sun. To determine the total number of stars within this sphere is one of the fundamental problems in stellar statistics, and to this question I shall return immediately.

The mean absolute magnitude of the near stars is distributed in the following way:—

M	0	1	3	4	5	6	7	8	9	10	11	12	13
Number ..	1	2	1	2	1	2	1	4	4	1	1	1	1.

What is the absolute magnitude of the near stars that are not contained in table? Evidently they must principally be faint stars. We may go further and answer that *all stars with an absolute magnitude brighter than* 6^m must be contained in this list. For if M is equal to 6 or brighter, m must be brighter than 6^m, if the star is nearer than one siriometer. But we have assumed that all stars apparently brighter than 6^m are known and are contained in the list. Hence also all stars *absolutely* brighter than 6^m must be found in table 5. We conclude that the number of stars having an absolute magnitude brighter than 6^m amounts to 8.

If, finally, the spectral type of the near stars is considered, we find from the last column of the table that these stars are distributed in the following way:—

Spectral type.....	B	A	F	G	K	M
Number	0	2	2	5	9	3.

For two of the stars the spectrum is for the present unknown.

We find that the number of stars increases with the spectral index. The unknown stars in the siriometer sphere belong probably, in the main, to the red types.

If we now seek to form a conception of the *total* number in this sphere we may proceed in different ways. EDDINGTON, in his "Stellar movements", to which I refer the reader, has used the proper motions as a scale of calculation, and has found that we may expect to find in all 32 stars in this sphere, confining ourselves to stars apparently brighter than the magnitude $9^m.5$. This makes 8 stars per cub. sir.

We may attack the problem in other ways. A very rough method, which, however, is not without importance, is the following. Let us suppose that the Galaxy in the direction of the Milky Way has an extension of 1000 siriometers and in the direction of the poles of the

Milky Way an extension of 50 sir. We have later to return to the fuller discussion of this extension. For the present it is sufficient to assume these values. The whole system of the Galaxy then has a volume of 200 million cubic siriometers. Suppose further that the total number of stars in the Galaxy would amount to 1000 millions, a value to which we shall also return in a following chapter. Then we conclude that the average number of stars per cubic siriometer would amount to 5. This supposes that the density of the stars in each part of the Galaxy is the same. But the sun lies rather near the centre of the system, where the density is (considerably) greater than the average density. A calculation, which will be found in the mathematical part of these lectures, shows that the density in the centre amounts to approximately 16 times the average density, giving 80 stars per cubic siriometer in the neighbourhood of the sun (and of the centre). A sphere having a radius of one siriometer has a volume of 4 cubic siriometers, so that we obtain in this way 320 stars in all, within a sphere with a radius of one siriometer. For different reasons it is probable that this number is rather too great than too small, and we may perhaps estimate the total number to be something like 200 stars, of which more than a tenth is now known to the astronomers.

We may also arrive at an evaluation of this number by proceeding from the number of stars of different apparent or absolute magnitudes. This latter way is the most simple. We shall find in a later paragraph that the absolute magnitudes which are now known differ between -8 and $+13$. But from mathematical statistics it is proved that the total range of a statistical series amounts upon an average to approximately 6 times the dispersion of the series. Hence we conclude that the dispersion (σ) of the absolute magnitudes of the stars has approximately the value 3 (we should obtain $\sigma = [13 + 8]:6 = 3.5$, but for large numbers of individuals the total range may amount to more than $6\,\sigma$).

As, further, the number of stars per cubic siriometer with an absolute magnitude brighter than 6 is known (we have obtained $8:4 = 2$ stars per cubic siriometer brighter than 6^m), we get a relation between the total number of stars per cubic siriometer (D_0) and the mean absolute magnitude (M_0) of the stars, so that D_0 can be obtained, as soon as M_0 is known. The computation of M_0 is rather difficult, and is discussed in a following chapter. Supposing, for the moment, $M_0 = 10$ we get for D_0 the value 22, corresponding to a number of 90 stars within a distance of one siriometer from the sun. We should then know a fifth part of these stars.

34. *Parallax stars.* In § 22 I have paid attention to the now available catalogues of stars with known annual parallax. The most extensive of these catalogues is that of WALKEY, containing measured parallaxes of 625 stars. For a great many of these stars the value of the parallax measured must however be considered as rather uncertain, and I have pointed out that only for such stars as have a parallax greater than 0″.04 (or a distance smaller than 5 siriometers) may the measured parallax be considered as reliable, as least generally speaking. The effective number of parallax stars is therefore essentially reduced. Indirectly it is nevertheless possible to get a relatively large catalogue of parallax stars with the help of the ingenious spectroscopic method of ADAMS, which permits us to determine the absolute magnitude, and therefore also the distance, of even farther stars through an examination of the relative intensity of certain lines in the stellar spectra. It may be that the method is not yet as firmly based as it should be,[1]) but there is every reason to believe that the course taken is the right one and that the catalogue published by ADAMS of 500 parallax stars in Contrib. from Mount Wilson, 142, already gives a more complete material than the catalogues of directly measured parallaxes. I give here a short resumé of the attributes of the parallax stars in this catalogue.

The catalogue of ADAMS embraces stars of the spectral types F, G, K and M. In order to complete this material by parallaxes of blue stars I add from the catalogue of WALKEY those stars in his catalogue that belong to the spectral types B and A, confining myself to stars for which the parallax may be considered as rather reliable. There are in all 61 such stars, so that a sum of 561 stars with known distance is to be discussed.

For all these stars we know m and M and for the great part of them also the proper motion μ. We can therefore for each spectral type compute the mean values and the dispersion of these attributes. We thus get the following table, in which I confine myself to the mean values of the attributes.

[1]) Compare ADAMS' memoirs in the Contributions from Mount Wilson.

TABLE 6.

MEAN VALUES OF m, M AND THE PROPER MOTIONS (μ) OF PARALLAX STARS OF DIFFERENT SPECTRAL TYPES.

Sp.	Number	m	M	μ
B	15	+ 2.03	— 1.67	0″.05
A	46	+ 3.40	+ 0.64	0.21
F	125	+ 5.60	+ 2.10	0.40
G	179	+ 5.77	+ 1.68	0.51
K	184	+ 6.17	+ 2.31	0.53
M	42	+ 6.02	+ 2.30	0.82

We shall later consider all parallax stars taken together. We find from table 6 that the apparent magnitude, as well as the absolute magnitude, is approximately the same for all yellow and red stars and even for the stars of type F, the apparent magnitude being approximately equal to $+ 6^m$ and the absolute magnitude equal to $+ 2^m$. For type B we find the mean value of M to be $- 1^m.7$ and for type A we find $M = + 0^m.6$. The proper motion also varies in the same way, being for F, G, K, M approximately 0″.5 and for B and A 0″.1. As to the mean values of M and μ we cannot draw distinct conclusions from this material, because the parallax stars are selected in a certain way which essentially influences these mean values, as will be more fully discussed below. The most interesting conclusion to be drawn from the parallax stars is obtained from their distribution over different values of M. In the memoir referred to, ADAMS has obtained the following table (somewhat differently arranged from the table of ADAMS),[1]) which gives the number of parallax stars for different values of the absolute magnitude for different spectral types.

A glance at this table is sufficient to indicate a singular and well pronounced property in these frequency distributions. We find, indeed, that in the types G, K and M the frequency curves are evidently resolvable into two simple curves of distribution. In all these types we may distinguish between a bright group and a faint group. With

[1]) The first line gives the stars of an absolute magnitude between — 4.9 and — 4.0, the second those between — 3.9 and — 3.0, &c. The stars of type B and A are from WALKEY's catalogue.

TABLE 7.

DISTRIBUTION OF THE PARALLAX STARS OF DIFFERENT SPECTRAL TYPES OVER DIFFERENT ABSOLUTE MAGNITUDES.

M	B	A	F	G	K	M	All
− 4	..	..	..	..	..	1	..
− 3	..	..	..	..	..	..	..
− 2	1	4	1	7	..	2	15
− 1	2	7	7	28	15	4	63
− 0	3	10	6	32	40	10	91
+ 0	1	11	6	7	14	11	50
+ 1	1	3	20	9	4	1	38
+ 2	..	5	48	26	..	1	80
+ 3	..	1	32	36	2	..	71
+ 4	..	1	5	25	25	..	56
+ 5	..	1	..	6	25	..	32
+ 6	..	2	..	3	10	..	15
+ 7	..	1	..	..	14	..	15
+ 8	..	..	..	..	3	7	10
+ 9	..	..	..	..	2	4	6
+ 10	..	..	..	..	..	..	..
+ 11	..	..	..	.	..	1	1
Total	8	46	125	179	154	42	554

a terminology proposed by HERTZSPRUNG the former group is said to consist of *giant* stars, the latter group of *dwarf* stars. Even in the stars of type F this division may be suggested. This distinction is still more pronounced in the graphical representation given in figures (plate IV).

In the distribution of all the parallax stars we once more find a similar bipartition of the stars. Arguing from these statistics some astronomers have put forward the theory that the stars in space are divided into two classes, which are not in reality closely related. The one class consists of intensely luminous stars and the other of feeble stars, with little or no transition between the two classes. If the parallax stars are arranged according to their apparent proper motion, or even according to their absolute proper motion, a similar bipartition is revealed in their frequency distribution.

Nevertheless the bipartition of the stars into two such distinct classes must be considered as vague and doubtful. Such an *apparent* bipartition is, indeed, necessary in all statistics as soon as individuals are selected from a given population in such a manner as the parallax stars are selected from the stars in space. Let us consider three attributes, say A, B and C, of the individuals of a population and suppose that the attribute C is *positively* correlated to the attributes A and B, so that to great or small values of A or B correspond respectively great or small values of C. Now if the individuals in the population are statistically selected in such a way that we choose out individuals having great values of the attributes A and small values of the attribute B, then we get a statistical series regarding the attribute C, which consists of two seemingly distinct normal frequency distributions. It is in like manner, however, that the parallax stars are selected. The reason for this selection is the following. The annual parallax can only be determined for near stars, nearer than, say, 5 siriometers. The direct picking out of these stars is not possible. The astronomers have therefore attacked the problem in the following way. The near stars must, on account of their proximity, be relatively brighter than other stars and secondly possess greater proper motions than those. Therefore parallax observations are essentially limited to (1) bright stars, (2) stars with great proper motions. Hence the selected attributes of the stars are m and μ. But m and μ are both positively correlated to M. By the selection ot stars with small m and great μ we get a series of stars which regarding the attribute M seem to be divided into two distinct classes.

The distribution of the parallax stars gives us no reason to believe that the stars of the types K and M are divided into the two supposed classes. There is on the whole no reason to suppose the existence at all of classes of giant and dwarf stars, not any more than a classification of this kind can be made regarding the height of the men in a population. What may be statistically concluded from the distribution of the absolute magnitudes of the parallax stars is only that the *dispersion* in M is increased at the transition from blue to yellow or red stears. The filling up of the gap between the "dwarfs" and the "giants" will probably be performed according as our knowledge of the distance of the stars is extended, where, however, not the annual parallax but other methods of measuring the distance must be employed.

Regarding the absolute brightness of the stars we may draw some conclusions of interest. We find from table 7 that the absolute magnitude

TABLE 8.

THE ABSOLUTELY FAINTEST STARS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Name	Position				Distance		Motion		Magnitude		Spectrum	
		$(\alpha\,\delta)$	Square	l	b	π	r	μ	W	m	M	Sp	m'
1	Proxima Centauri	(142262)	GD_{10}	281°	— 2°	0″.780	sir. 0.26	3″.85	sir./st. ..	11^m.0	+ 13^m.9	..	m' 13.5
2	van Maanens star	(004304)	GE_8	92	— 58	0.216	0.84	3.01	..	12.3	+ 12.7	F0	12.9
3	Barnards star	(175204)	GC_{12}	358	+ 12	0.515	0.40	10.29	— 19	9.7	+ 11.7	Mb	8.9
4	17 Lyræ C	(190332)	GC_2	31	+ 10	0.128	1.60	1.75	..	11.3	+ 10.3	..	12.5
5	C. Z. 5^h.243	(050744)	GE_7	218	— 35	0.319	0.65	8.75	+ 51	9.2	+ 10.1	K2	10.6
6	Gron. 19 VIII 234	(161839)	GB_1	29	+ 44	0.162	1.27	0.12	..	10.3	+ 9.8	..	..
7	Oe. A. 17415.........	(173768)	GB_3	65	+ 32	0.268	0.77	1.30	..	9.1	+ 9.7	K	10.5
8	Gron. 19 VII 20	(162148)	GB_2	41	+ 43	0.133	1.55	1.22	..	10.5	+ 9.6	..	..
9	Pos. Med. 2164	(184159)	GC_2	56	+ 24	0.292	0.71	2.28	..	8.9	+ 9.6	K	10.3
10	Krüger 60	(222457)	GC_8	72	0	0.256	0.81	0.94	..	9.2	+ 9.6	K5	10.8
11	B. D. + 56° 532	(021256)	GD_8	103	— 4	0.195	1.06	..	..	9.5	+ 9.4	..	..
12	B. D. + 55° 581	(021356)	GD_8	103	— 4	0.185	1.12	..	..	9.4	+ 9.2	G5	10.2
13	Gron. 19 VIII 48	(160438)	GB_1	27	+ 46	0.091	2.27	0.12	..	11.1	+ 9.3	..	..
14	Lal. 21185	(105736)	GB_5	153	+ 66	0.403	0.51	4.77	— 18	7.6	+ 9.1	Mb	8.9
15	Oe. A. 11677.........	(111466)	GB_8	103	+ 50	0.198	1.04	3.03	..	9.2	+ 9.1	Ma	11.0
16	Walkey 653	(155359)	GB_2	57	+ 45	0.175	1.18	..	..	9.5	+ 9.1	..	..
17	Yerkes parallax star ..	(021243)	GD_8	107	— 16	0.045	4.58	..	..	12.4	+ 9.1	..	..
18	B. D. + 56° 537.......	(021256)	GD_8	103	— 4	0.175	1.18	..	..	9.4	+ 9.0	..	..
19	Gron. 19 VI 266	(062084)	GC_8	97	+ 27	0.071	2.80	0.09	..	11.8	+ 9.0	..	.
	Mean...	..	..	..	27°.5	0″.244	sir. 0.99	2″.96	sir./st. 29.3	10^m.0	+ 9^m.9	K1	m' 10.9

of the parallax stars varies between —4 and +11, the extreme stars being of type M. The absolutely brightest stars have a rather great distance from us and their absolute magnitude is badly determined. The brightest star in the table is Antares with $M = -4.6$, which value is based on the parallax $0''.014$ found by ADAMS. So small a parallax value is of little reliability when it is directly computed from annual parallax observations, but is more trustworthy when derived with the spectroscopic method of ADAMS. It is probable from a discussion of the *B*-stars, to which we return in a later chapter, that the absolutely brightest stars have a magnitude of the order -5^m or -6^m. If the parallaxes smaller the $0''.01$ were taken into account we should find that Canopus would represent the absolutely brightest star, having $M = -8.17$, and next to it we should find RIGEL, having $M = -6.97$, but both these values are based on an annual parallax equal to $0''.007$, which is too small to allow of an estimation of the real value of the absolute magnitude.

If on the contrary the *absolutely faintest* stars be considered, the parallax stars give more trustworthy results. Here we have only to do with near stars for which the annual parallax is well determined. In table 8 I give a list of those parallax stars that have an absolute magnitude greater than 9^m.

There are in all 19 such stars. The faintest of all known stars is INNES' star "Proxima Centauri" with $M = 13.9$. The third star is BARNARD's star with $M = 11.7$, both being, together with α Centauri, also the nearest of all known stars. The mean distance of all the faint stars is 1.0 sir.

There is no reason to believe that the limit of the absolute magnitude of the faint stars is found from these faint parallax stars:—Certainly there are many stars in space with $M > 13^m$ and the mean value of M, for all stars in the Galaxy, is probably not far from the absolute value of the faint parallax stars in this table. This problem will be discussed in a later part of these lectures.

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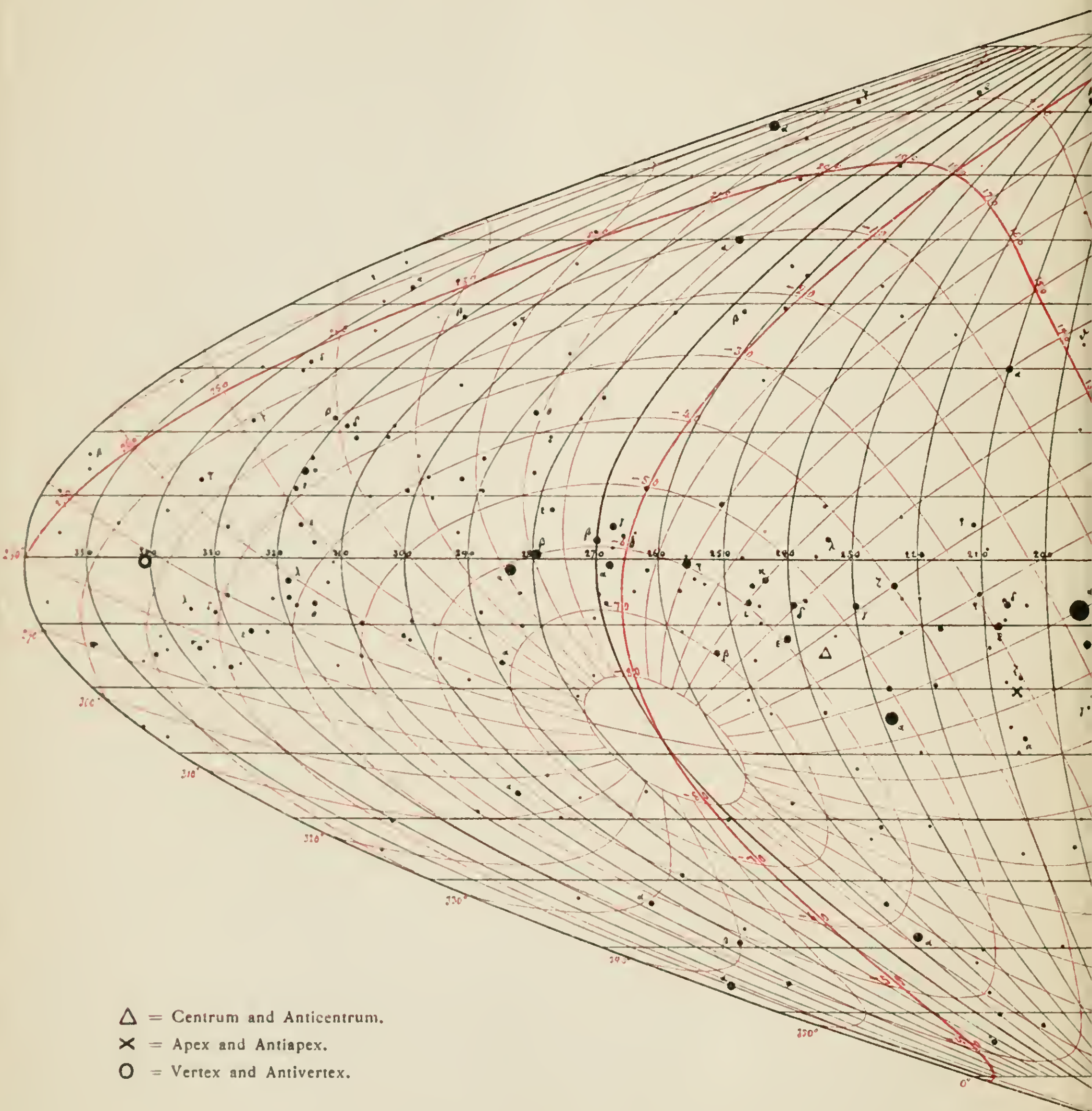

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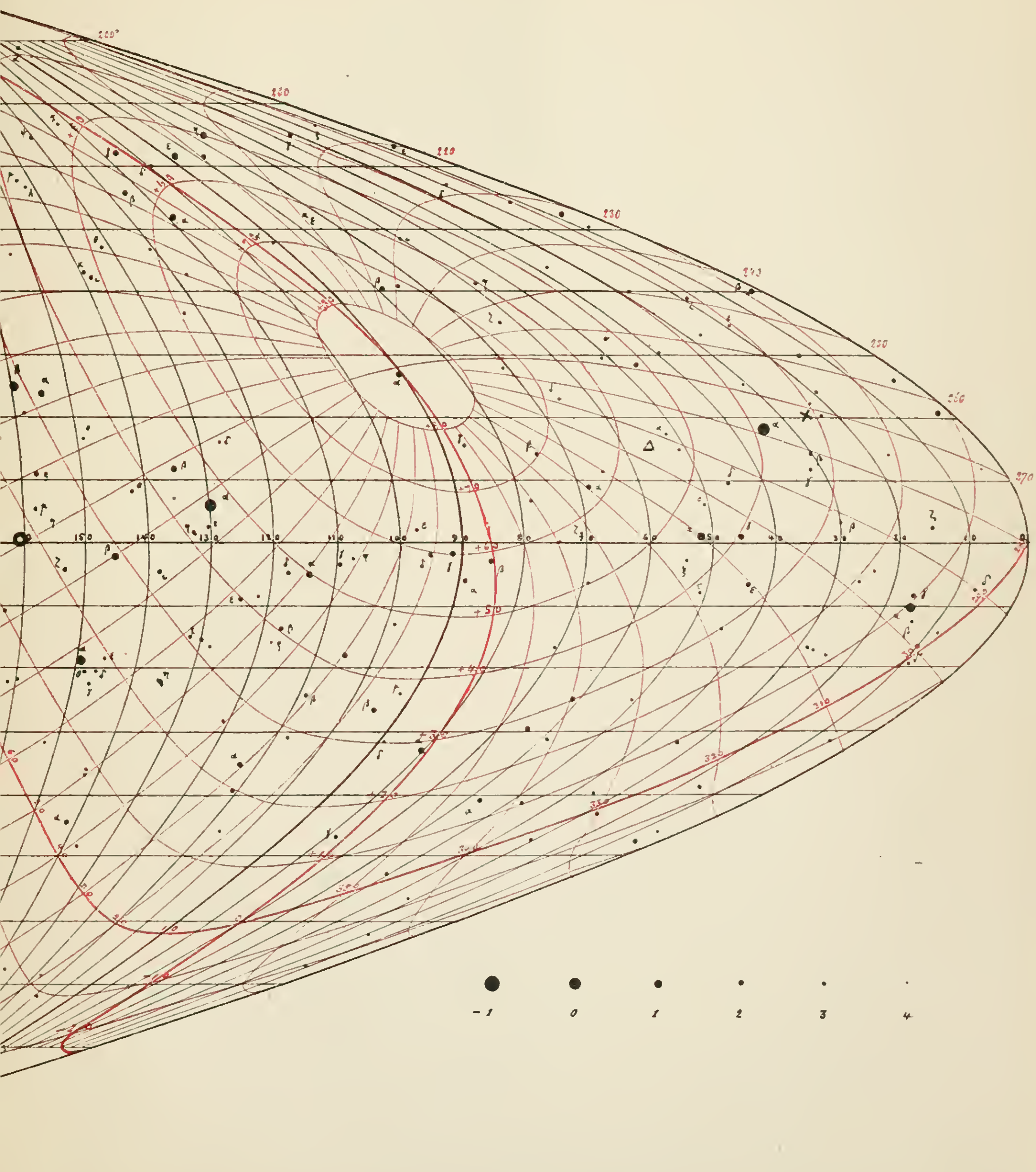

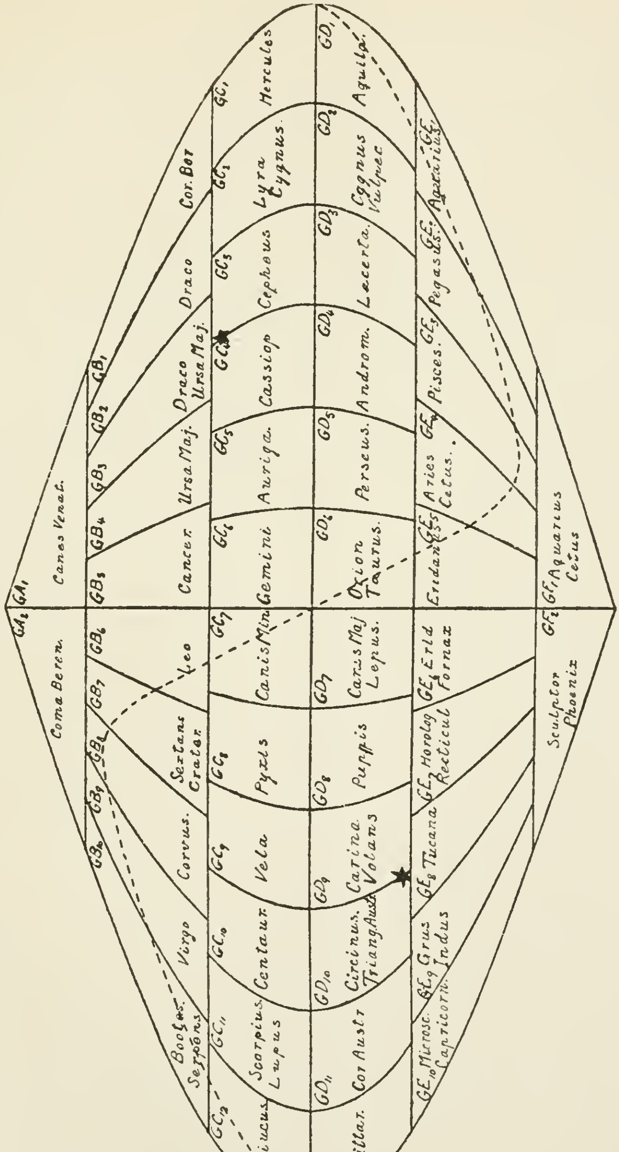

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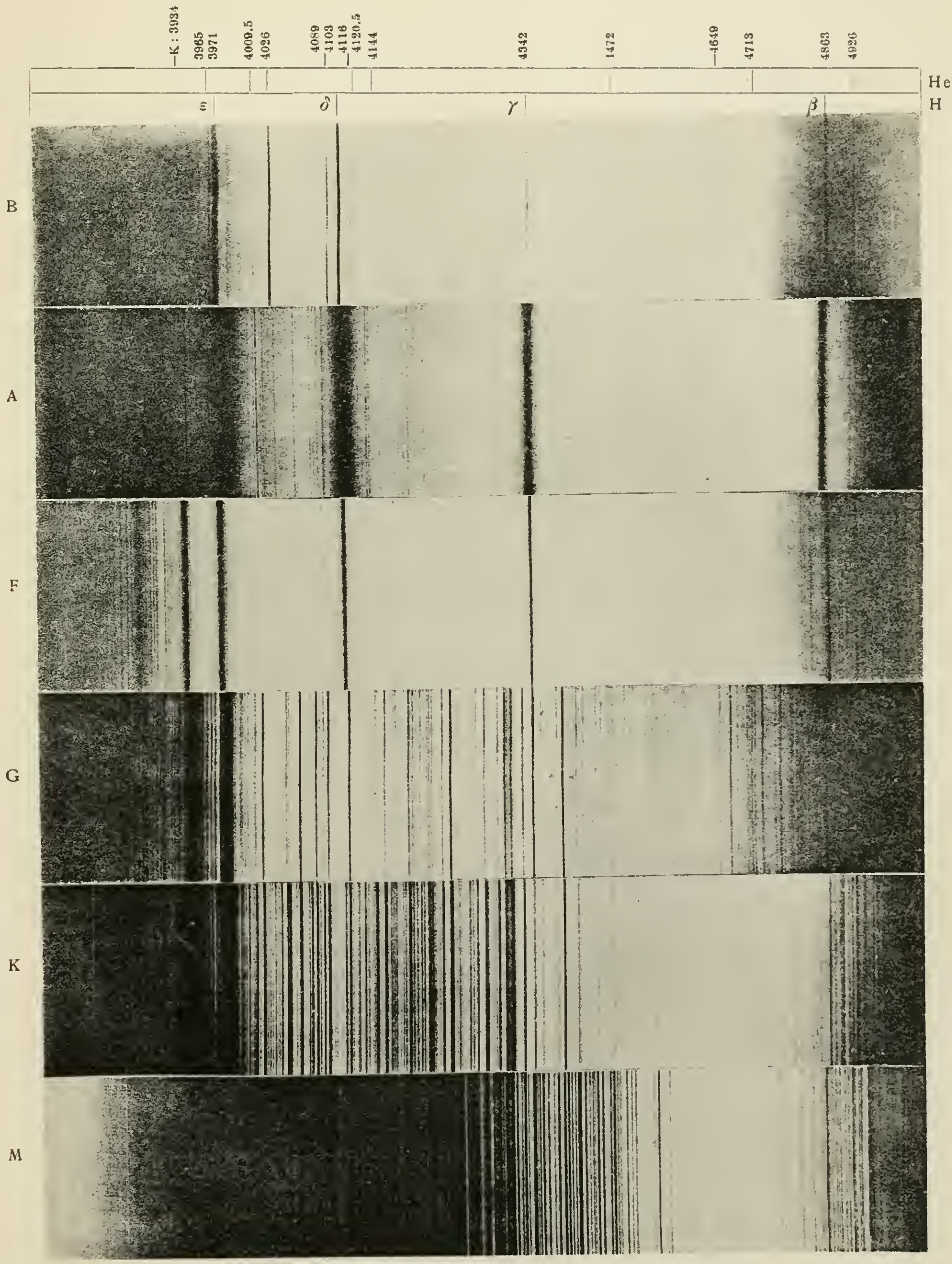
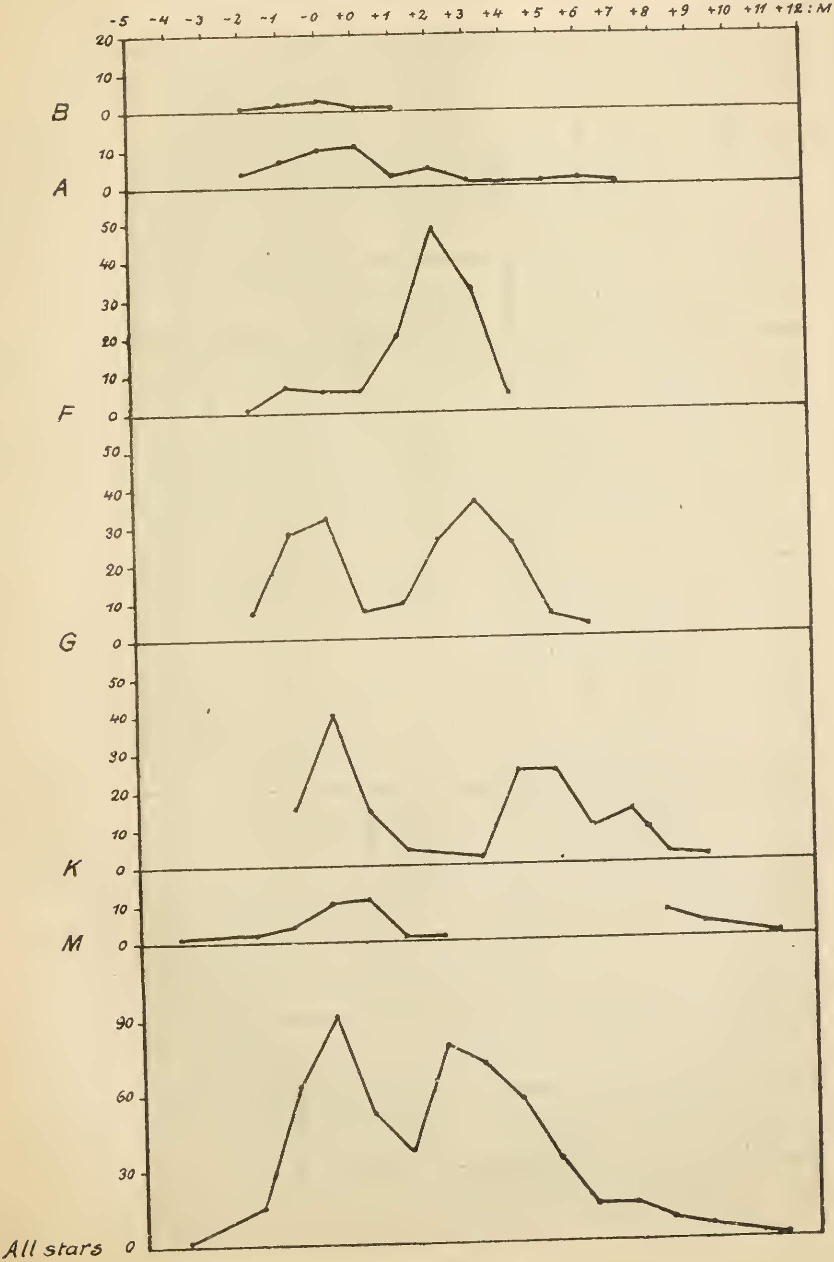


The Harvard Classification of Stellar Spectra.

Distribution of the parallax stars over different absolute magnitudes.



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